GRAPHIC
THERMODYNAMICS

HUNTINGTON

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BY

W. S. HUNTINGTON

PREFACE

THE chief purpose of the writer is to present a graphical method for the analysis of problems involving the heat equivalent of mechanical energy, particularly the varied problems relating to the compression and expansion of air, gases, and vapors. As an introduction to the graphical work, it has been found necessary to present, with all brevity possible, some of the general principles of thermodynamics. No originality or completeness is claimed for the subject matter of that chapter; the intention is to present only those subjects essential to the study in hand.

In addition to an acquaintance with the basic principles of thermodynamics, it is essential that the reader be familiar with the use of the Mollier chart as made for steam and ammonia vapors and with the tables of properties of those vapors. Throughout the work the reference to steam tables or charts is to the work of Marks and Davis (*Steam Tables and Diagrams.* Longmans, Green) and any reference to ammonia tables or charts is to those published by the U.S. Bureau of Standards, (Circular No. 142, *Tables of Thermodynamic Properties of Ammonia*). The total heat charts presented herein are not intended to replace those found in the publications named, but to explain their use in connection with the graphical analyses discussed in these pages. The charts for internal energy have been constructed by the writer, the basic data being taken from the tables mentioned. Permission has been obtained, in each instance, for the use made of the publication named.

The graphical method permits one to scale from a diagram the heat equivalent of the work done in the cylinder of a compressor or an engine. The heat-work diagram is simple and easily constructed, requiring only the use of pencil, scale and straight-edge, and reference to the charts contained herein or others of like character. The results obtained agree closely with those obtained by use of the well-known indicator diagram; the latter requires careful computation of the area of an irregular figure, whereas the heat-work diagram only requires the scaling of the length of a straight line.

The writer believes the graphical treatment to be new, and in order to show its utility, he has applied it in the solution of a variety of practical problems. Moreover, it has seemed necessary to consider the entire range of pressures and temperatures, from absolute zero on one hand to infinity on the other, in addition to practicable working conditions. This has led to presentation of the general graphs, accompanied by brief discussions, which it is believed will be found of interest and value.

No attempt has been made to deal with the details of construction or operation of compressors, engines, and pneumatic tools. Existing books cover these subjects thoroughly. It has been the aim of the writer to place in the hands of those interested a useful tool — the heat-work diagram — something every one may apply to the solution of his own problems, whether of design, detail, or operation. It is offered to the engineer and the engineering student in the hope that it may be useful.

W. S. HUNTINGTON

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CHAPTER I

HEAT AND WORK

Heat. — There is no complete and satisfactory definition of the energy or activity familiarly known to us as heat. The hypothesis that has earned general acceptance regards heat as a mode or manifestation of motion. The molecules of which any substance is composed are believed to be in continual vibratory motion, the rapidity of the vibrations increasing with an increase in heat. We shall study the effects caused by adding heat to or subtracting it from certain substances, especially the common gases and vapors; and we can do that, happily, without first writing a definition of heat itself.

Temperature. — The temperature of a substance is, in general, the measure of the heat that it contains, measured above an assumed datum or level (vapors excepted). A convenient datum is the constant temperature at which pure water freezes when at the atmospheric pressure prevailing at sea level. This determines the point marked as zero on the centigrade thermometer scale or that marked 32 on the scale of the ordinary Fahrenheit thermometer. A second nature-fixed point is required, and one is offered by the boiling point of pure water, also at sea-level atmospheric pressure. This unvarying temperature determines the point marked 100° on the Centigrade, or 212° on the Fahrenheit thermometer scale. Between these two nature-fixed points the Centigrade scale is divided into 100 equal parts or the Fahrenheit scale into 180 equal parts, each part being called one degree. All temperatures written in this book are to Fahrenheit scale.

Sensible Heat. — Heat which shows its presence in such manner that any increase or decrease may be measured by a

thermometer (or a pyrometer) is commonly called "sensible" heat; it is that form or effect of heat manifest to the senses of the body; that which we all know as heat. If it be withdrawn from a substance in large measure, that substance becomes "cold."

Absolute Zero Temperature. — The mind cannot imagine a state of matter wherein no heat would be present. However, analysis of the action of gases when subjected to changes in pressure, temperature, and volume indicates that there is a third nature-fixed point, one that for practical purposes may be regarded as the absolute zero of temperature, it being at 459.6 degrees below the zero of the Fahrenheit scale (or at 273.1° below the zero of the Centigrade scale). If the temperature of some substance as shown by a thermometer be, say 47°, its absolute temperature is $47 + 459.6 = 506.6°$; if $- 37°$ by thermometer, the absolute temperature is 422.6°, etc. The law determining this zero point is stated and discussed in a later paragraph (page 9).

Heat Exchange. — If two or more substances having different temperatures be placed near together or in contact, their temperatures will tend to equalize. Heat will flow from the warmer to the colder substance. If a group or system of substances could be insulated completely from the influence of all others, then in due time the temperatures of all members of the group would be found substantially equal. Substances differ, however, in their heat-receptive capacity. Thus the quantity of heat which would suffice to raise the temperature of a pound of one substance through ten degrees might suffice to raise the temperature of a pound of some other substance only five degrees; that of a pound of a third substance four degrees; that of a fourth twenty degrees, etc. The quantity of heat transferred would be the same in the four instances, but its effect as manifested by temperature changes would be different.

Heat Unit. — It is convenient to have a fixed unit for the measurement of heat content and heat transfer. Such in English-speaking countries is the British thermal unit, written

for brevity as the "B.t.u." One B.t.u. is the measure of the quantity of heat required to raise the temperature of one pound of pure water through one degree, Fahrenheit. Authorities differ as to the temperature at which this should be measured, as the quantity required is not constant per degree for the entire range between the freezing and boiling points. The variation is small, however, and may be ignored for our present purposes.

Specific Heat. — The quantity of heat which would raise the temperature of one pound of water through one degree would, if applied to a pound of copper, raise its temperature through more than ten degrees; or, conversely, it requires less than one-tenth as much heat to raise the temperature of a pound of copper one degree as to raise that of a pound of water one degree, the exact quantity being 0.094 B.t.u. This ratio is the "specific heat" of copper. There is a definite, determinable specific heat value for every substance, those for the common materials being tabulated in several of the engineering handbooks. For any substance, the specific heat is the ratio between the heat necessary for raising the temperature of one pound of it one degree and the heat necessary for raising the temperature of a pound of pure water one degree.

Heat-Change Calculations. — If the specific heat value for a material and its weight in pounds are known, the quantity of heat required to warm it through any specified number of degrees of temperature is the product of its weight in pounds multiplied by its specific heat per pound and by the number of degrees of temperature increase. Similarly, for cooling, the heat that must be removed from a body to cool its temperature through a given number of degrees is the product of its weight in pounds multiplied by its specific heat per pound and by the number of degrees of temperature fall. As an example, suppose that a piece of cast iron weighing 120 pounds is to be heated from initial temperature of 67° up to 324°. The specific heat for cast iron, as given in the handbooks, is 0.127 B.t.u. per pound, and therefore the heat required is

$$120 \times 0.127 \times (324 - 67) = 3916.7 \ B.t.u.$$

As an example in cooling, let a mass of stone weighing 2 tons cool from temperature 92° down to 69°, the specific heat of the stone being 0.22 B.t.u. per pound. The heat loss is

$$4000 \times 0.22 \times (92 - 69) = 20{,}240 \; B.t.u.$$

If this quantity of heat could be applied in some manner to heat a certain definite weight of air, say 30,000 pounds, the specific heat of air being taken, for the moment, as 0.24 B.t.u. per lb. the temperature rise for the air would be

$$\frac{20{,}240}{0.24 \times 30{,}000} = 2.81 \; degrees.$$

Solids, Liquids, and Gases. — We are familiar with these three states of matter. In the solid state the forces which cause the molecules to cohere are sufficient to withstand all external forces tending to disruption or distortion, so that the mass may be lifted, moved about, and acted upon by comparatively powerful forces without any perceptible change in shape. In the liquid form the molecules of the substance are perhaps slightly farther apart than in the solid, and the force holding them together is not adequate to prevent change of shape or division of the mass by moderate external forces, such as that of gravity; hence a liquid may be poured, flowed, or sprayed at will. In the gaseous state the molecules of the substance have become separated from one another by relatively great distances which tend to increase with further reception of heat or with enlargement of the container in which a given weight of gas is confined. Change of state may be caused by a change in heat content. Thus the reception of a certain quantity of heat per pound by a solid apparently breaks the bond or attraction between its constituent molecules to the extent that they become free to move upon one another, accompanied in general by a slight increase in the volume occupied by a given weight. The liquid thus formed may be heated up to a certain temperature without further change in state, but eventually a point is reached beyond which any further reception of heat causes the molecules of the substance to separate violently from one

another and hence to occupy a much greater volume, this being the gaseous or vapor state.

Linear Expansion Due to Heat. — When an unconfined mass of a substance is heated, its constituent molecules separate a little, one from another, the distance varying with the substance and with the quantity of heat received per unit of weight. In the solid and the liquid states the expansion is very little, relatively, but is great in the gaseous state. If the substance be confined in such manner that its volume cannot increase, the reception of heat causes an increase in the pressure exerted by it against the interior surfaces of the walls of the container. A material will expand through a certain definite proportion of its length or volume for every degree increase in temperature, or for every unit of heat received. For instance, in the common thermometer the mercury rises in the stem a certain markable distance for every degree rise of its own temperature.

Work. — Work is done whenever a body is moved against resistance. Work is measured by the "foot-pound" unit. One foot-pound is the work equivalent to that of raising a weight of one pound through a distance of one foot. If the pound be raised 3 feet, the work done is 3 foot-pounds; if 3 pounds be raised one foot, the work done is 3 foot-pounds; if 100 pounds be raised 10 feet, the work done is 1000 foot-pounds; if a weight of 12 pounds be lifted 9 inches, the work done is 9 foot-pounds; etc. The " inch-pound " could be used as the unit of work measurement, as indeed it is in certain computations, but in general the foot-pound is more convenient. If a gas entering a cylinder against a piston having a face area of one square foot moves that piston through a distance of 18 inches against a pressure of 8000 pounds, the work done by the gas is

$$8000 \times 1\tfrac{1}{2} = 12,000 \ \textit{foot-pounds or}$$
$$8000 \times 18 = 144,000 \ \textit{inch-pounds.}$$

It may be noted that the element of time is not involved in the definition of work.

Power. — A unit for power measurement must involve the time element. The common unit is the horse-power, or H.P., which is the equivalent of 33,000 foot-pounds of work done in one minute of time or 550 foot-pounds of work in one second of time. Thus, as in the example above given, if 12,000 foot-pounds of work be done in three seconds of time, the work is at the rate of

$$\frac{12,000}{3 \times 550} = 7.272 \ H.P.$$

Mechanical Equivalent of Heat. — Mechanical energy, expended in the production of work, is convertible into heat energy. Heat energy, to a certain extent, is convertible into mechanical energy for the production of work. The energy represented by one B.t.u. of heat is the equivalent of the energy required for doing 777.5 foot-pounds of work. This quantity is the "mechanical equivalent of heat." It follows that one horse-power is the mechanical equivalent of

$$\frac{33,000}{777.5} = 42.45 \ B.t.u. \ of \ heat \ energy \ expended \ per \ minute.$$

Heat of Fusion, Solidification. — When a solid substance is heated, the reception of heat causes a rise in temperature up to a certain point, the melting point, the heat received per degree of temperature rise agreeing with the specific heat value of the substance. After the melting point is reached, the temperature remains stationary while the change of state from solid to liquid is in progress. From the instant when fusion of the mass is complete, the temperature again will rise with continued application of heat. In solidification the process is reversed. Withdrawal of heat from the hot liquid proceeds until the fusion or melting temperature is reached; then there is no further drop in temperature until the change to solid form is complete. The heat received during the melting process at constant temperature, or the equal amount given out while solidification is in progress, is the "latent heat of fusion" of the substance considered.

Heat of Vaporization, Condensation. — In general, when a liquid is heated, the reception of heat causes a rise in temperature up to a certain point, the location of which varies with the pressure on the liquid. For example, at atmospheric pressure a pound of water receives heat at the rate of one B.t.u. (very nearly) per degree temperature rise up to its boiling or vaporization point, which is 212°. Once that temperature is attained the pound will absorb about 970 B.t.u. of heat additional while passing from liquid to vapor form, the temperature remaining stationary at 212° while the change is in progress. The temperature cannot be made to rise higher than 212° as long as the pressure remains constant as stated. If the pressure on the water and its rising vapor becomes 20 lbs. per square inch, the temperature must rise to 228° before further vaporization takes place, and then only about 960 B.t.u. of heat will be required per pound to effect the change. The heat thus required for conversion of liquid to vapor after the vaporization temperature is attained is the "latent heat of vaporization" for the liquid considered. Its amount varies, being less for high than for low pressures. When the process is reversed and the substance changes back from the vapor to the liquid form, it must yield to some other substance or substances an amount of heat exactly equal to that it received in vaporization, if at the same pressure.

Wet, Dry, and Superheated Vapors. — When vaporization obtains in an industrial apparatus such as a steam boiler or a refrigerating coil, it proceeds with more or less violence. The freed molecules which constitute the vapor do not part gently and individually from the body of liquid, but force their way through it in mass form, or bubbles, and carry with them into the vapor body more or less of the liquid in the shape of fine floating particles. Heat is needed to vaporize these particles, but this heat cannot be abstracted from the surrounding vapor, which is at the same temperature. If, then, the vapor with liquid particles held in suspension be separated from the heat source, as when steam flows out from a boiler and through a steam pipe toward an engine cylinder, the liquid particles

will remain liquid. Such vapor is known as " wet " vapor, while
one without any floating particles of unvaporized liquid, but
at the vaporizing temperature, is known as a " dry " or " satu-
rated " vapor.   Referring to wet vapor, the percentage of a
given weight of the material which is vapor is known as its
" quality."   For example, if 3 per cent be unvaporized, floating
along in suspension in the body of vapor, then the quality is
97 per cent, or, as often written, it is 0.97. When removed to
a sufficient distance from the liquid from which it has risen, a
vapor may be heated to a temperature higher than that at
which it formed, and this without increase of pressure if the
conditions are such that the volume is free to increase as heat
is received. For example, steam forming at 20 lbs. absolute
pressure and 228° temperature cannot be heated hotter than
228° as long as it remains in contact with the boiling water
from which it is rising; but if removed therefrom, placed in a
separate container or even in a communicating pipe or super-
heater at some distance from the boiler, it may there be heated
to any practicable high temperature, say to 400°, 500°, etc.
A vapor in this condition is said to be superheated. When wet
vapor passes into a superheater, the heat necessary to complete
its vaporization is abstracted from the surrounding superheated
vapor.

Gases. — A gas may be considered as a superheated vapor,
far removed from the conditions which induced or permitted
its formation. A " perfect " or " ideal " gas is one which at
all pressures and temperatures would behave in agreement
with the mathematical theory proved true for matter in per-
fect gaseous state. Practically all gases can be liquefied, and
hence none is to be considered truly perfect; but some are
practically so for the usual working conditions. Among these
are oxygen, hydrogen, and air.

Boyle's Law. — **" The volume of a perfect gas, at constant
temperature, varies inversely as the pressure."** For example,
if a pound of a certain gas occupies 10 cu. ft. when under
pressure of 30 lbs. per square inch, absolute, at the same tem-
perature and 60 lbs. pressure it would occupy only 5 cu. ft.;

at 120 lbs., $2\frac{1}{2}$ cu. ft.; at 20 lbs., 15 cu. ft.; etc. That is, at any given temperature the product of pressure and volume is a constant number. In the example given, $30 \times 10 = 300$; $60 \times 5 = 300$; etc. At any other temperature the product would be some other constant number. (Boyle, England, 1627–1691. Also credited to Mariotte, France, 1620–1684.)

Isothermal Change. — When a pressure and volume change takes place without change in temperature, as in the example of preceding paragraph, it is an "isothermal" change, the word "isothermal" meaning, in this case, "having equal temperatures." Obviously, Boyle's law applies in cases of isothermal change. The following symbols are convenient:

V_e = *volume of one pound of gas in expanded state, before compression, at a certain temperature, and*

V_c = *volume of the same pound after compression to some higher pressure, the temperature remaining constant; also,*

P_e = *pressure on the gas before compression, per square inch or per square foot of container, cylinder, or piston surface;*

P_c = *pressure after the compression.*

Then, according to Boyle's law, $V_e P_e = V_c P_c$. (These two products being equal to the same constant are equal to each other.) Therefore, it follows that:

$$V_c = \frac{V_e P_e}{P_c}; \quad P_c = \frac{V_e P_e}{V_c}; \quad V_e = \frac{V_c P_c}{P_e}; \text{ and } P_e = \frac{V_c P_c}{V_e}.$$

Boyle's law applies with fair accuracy to the nearly perfect gases, such as oxygen, nitrogen, and air, but only approximately to the vapors, such as steam, ammonia, carbon dioxide, etc.

Charles's Laws. — These laws have been stated by a well-known authority as follows:

(1) **"Under constant pressure an ideal gas will increase in volume equally for every degree increase in absolute temperature."**

(2) **"Under constant volume an ideal gas will increase in pressure equally for every degree increase in absolute temperature."**

(Charles, France, 1746–1823. Also known as the laws of Gay-Lussac, France, 1778–1850.)

It has been learned experimentally that the expansion of an ideal gas, at atmospheric pressure, is for every degree rise in absolute temperature 1/492 part of the volume occupied by the gas as when at 32° temperature. For example, if the volume per pound at 32° and atmospheric pressure be, say, 12 cu. ft., and sufficient heat be applied to raise the temperature to 33°, the pressure the while being held constant, the volume increase will be

$$\frac{12}{492} = 0.02439 \ cubic \ feet,$$

the volume as at 33° thus becoming $12 + 0.02439 = 12.02439$ cu. ft. If the gas be heated through 10 degrees additional to 43° temperature, the further increase in volume will be 0.2439 cu. ft. If the temperature be raised to $32° + 492°$ $= 524°$, the volume will be doubled; that is, it will be increased by 492/492 of the original volume and thus become 24 cu. ft. In the other direction, starting at 32° and cooling to 31°, the shrinkage will be 0.02439 cu. ft.; if cooled through 100 degrees down to $-68°$, the shrinkage will be 2.439 cu. ft., etc. Continuing in this direction in strict accordance with Charles's law, it appears that if the temperature could be made to fall through 492 degrees down to $-460°$, the shrinkage would be 492/492 of the original volume and that hence the final volume would be nil. Our knowledge of the nature of matter hardly permits us to believe that it could be made to vanish thus, even if it were possible to reduce its temperature down to $-460°$. As a matter of fact, all gases thus far tested condense to the liquid state at a temperature above that point. It is sufficient for our purposes to know that the so-called perfect gases follow the law closely within practicable limits of pressures and temperatures. The round number $-460°$ is sufficiently exact for most purposes; but where greater accuracy is necessary, the accepted value for the absolute zero is $-459.6°$, Fahrenheit, as stated.

Zero of Pressure.   Perfect Vacuum. — According to Charles's second law, the pressure of an ideal gas in a container of constant volume increases equally for each degree rise in its temperature.   This is equivalent to saying that pressure varies with the absolute temperature; and with reasoning parallel to that of the preceding paragraph, it follows that if the temperature of a fixed volume of gas be raised from, say, 32° (492° absolute) to 524° (984° absolute) the rise being 492 degrees, the pressure will be doubled.   Likewise, on the other hand, it appears that, if the temperature could be reduced to the absolute zero, all pressure would be removed, and we should thus have a perfect vacuum.   Although not attainable in practice, this furnishes a convenient level or datum from which to reckon pressures and is known as the absolute zero of pressure, corresponding to the absolute zero of temperature.   Atmospheric pressure is, for practical purposes, 14.7 lbs. per square inch absolute if measured at or near the sea level, decreasing with increase of altitude.

Pressure Gage. — The ordinary pressure gage, as made for air and gas compressors, steam engines, etc., is so constructed that the pointer stands at the zero figure when the pressure inside the container or apparatus is equal to that of the air outside.   That, of course, may or may not be 14.7 lbs. per square inch, but for ordinary purposes the addition of 14.7 lbs. to the gage reading gives with sufficient exactness the corresponding absolute pressure.   Gages for partial vacuum are graduated from 0 to 30, based on the fact that air pressure at sea level is approximately equal to that of the weight of a column of mercury one square inch in cross section and 30 inches in height.   (Or 29.92 inches, if extreme accuracy be needed.)   A pressure of " two inches of mercury," then, means 2/30 of the atmospheric pressure of 14.7 lbs.   The gage, however, marks the difference between inches-of-mercury pressure and full atmospheric; that is, pressure equal to 2 inches of mercury is marked by the gage as 28 inches of vacuum; 5 inches vacuum means 30 − 5 = 25 inches of mercury, etc.   Converting to pounds of absolute pressure 5 inches of vacuum means,

$$\frac{(29.92 - 5) \times 14.7}{29.92} = 12.24 \; lbs. \; per \; square \; inch.$$

Ideal Gases. Pressure-Volume-Temperature Changes. —
According to Boyle's law, if P represents the pressure and V
the volume of one pound of an ideal gas at any temperature,
the product of PV = a constant. According to Charles's law,
if T represents an absolute temperature, V varies directly as
T varies; also P varies directly as T varies. It follows that,
combining, PV must vary as T varies, and hence we may
write

$$\frac{PV}{T} = a \; constant \; number.$$

This constant number may be represented by the symbol R.
Then, $\frac{PV}{T} = R$, or, transposed, $PV = RT$. Now it follows
that with any other set of values for pressure, volume, and
temperature, such as p, v, and t, for instance, the ratio of $\frac{pv}{t}$ = a
constant number; and, as when the value of T varies to the
value of t, the product of PV must vary likewise and take the
same value as pv, it must be true that the constant number is
identical for all such cases with any given gas. This gas con-
stant is commonly denoted in formulas and tables by the symbol
R as used above. Values for a number of the more common
gases are given in Table I. For vapors and imperfect gases
the values of R are not constant.

If the value for R be known, and any two of the quantities
P, V, T, the third is readily found, as we have

$$P = \frac{RT}{V}; \quad V = \frac{RT}{P}; \quad T = \frac{PV}{R}; \; and \; R = \frac{PV}{T}.$$

These equations hold true for one pound weight of the gas,
the pressure, P, being in pounds per square *foot* and volumes
in cubic feet. If the pressure be stated in pounds per square
inch, it must be accompanied by the multiplier 144 to convert
to pounds per square foot. To make the equations general,

applicable to any weight, W, of the gas, the basic equation becomes $PV = WRT$, and again the several transpositions follow as above. Thus,

$$P = \frac{WRT}{V}; \quad V = \frac{WRT}{P}; \quad T = \frac{PV}{WR}; \quad R = \frac{PV}{WT}; \text{ and, } W = \frac{PV}{RT}.$$

As an example in application of the above equations, take the case of oxygen gas, for which the constant, R, is 48.283. (See Table I.) Let it be desired to find the volume occupied by one pound of oxygen as at 163 lbs. per square inch absolute pressure and 120° temperature. Here we have

$$V = \frac{RT}{P} = \frac{48.283 \times 579.6}{163 \times 144} = 1.192 \text{ cubic feet.}$$

Specific Heat of Gases. — Let a pound of an ideal gas be heated until its temperature has risen one degree, its volume meanwhile being free to expand against a certain constant pressure and the extent of expansion being in strict agreement with Charles's law. A definite number of B.t.u., or fraction of one B.t.u. of heat, is required to effect the change in temperature and volume. This definite quantity of heat is the "specific heat at constant pressure" of the gas in question. Some portion of this required heat is transformed into mechanical energy and as such is expended in enlarging the volume against the pressure acting on the gas from without. If now a pound of the same gas be heated in a closed container so constructed that volume cannot increase, then no work is done in enlargement of volume, and less heat is required to raise the temperature through one or any number of degrees than if the volume were free to increase during the heating process. In this case the heat required to effect one degree temperature change is the "specific heat at constant volume" of the gas in question. The specific heat at constant pressure is commonly designated by the symbol C_p, that at constant volume by the symbol C_v.

The value of C_v for any gas being less than the value of C_p, it follows that the ratio of C_p to C_v always exceeds 1.00. Moreover, as for an ideal gas the values of C_p and C_v are constant,

their ratio is likewise constant. This ratio is commonly desig-
nated by the symbol y, or by γ (gamma, third letter of the
ancient Greek alphabet) or by k. In this book the symbol y
will be used. Then for any gas

$$y = \frac{C_p}{C_v}.$$

Values of the constant y for several gases are shown in Table I.
Values of y for several vapors also appear in that table, being
those which are true at atmospheric pressure and $32°$ tempera-
ture; the variations from the values shown may be quite
marked at other pressure and temperature conditions.

TABLE I

CONSTANTS FOR GASES AND VAPORS

Gas or Vapor	C_p	C_v	$C_p - C_v$	y	R
Air	0.2396	0.1711	.0685	1.400	53.294
Hydrogen..........	3.4100	2.4358	.9742	1.400	757.93
Oxygen...........	0.2173	0.1552	.0621	1.400	48.283
Nitrogen..........	0.2438	0.1740	.0698	1.400	54.192
Helium...........	1.250	0.753	.497	1.66	386.42
Ammonia..........	0.52	0.40	.12	1.30	....
Carbon Dioxide.....	0.2008	0.1548	.046	1.297	....
Sulphur Dioxide....	0.1544	0.1229	.0315	1.256	....

The difference between the two specific-heat values for a
gas is the heat equivalent of the mechanical energy spent in
enlargement of volume when one pound of the gas is heated
through one degree of temperature at constant pressure. As
an example, and for proof as well, we take a pound of oxygen
gas, which at $32°$ temperature and atmospheric pressure oc-
cupies 11.2233 cubic feet volume. Let this be heated one
degree, to $33°$, the pressure remaining constant; then according
to Charles's law the volume increase will be

$$\frac{11.2233}{492} = 0.02281 \ cubic \ feet.$$

Let the reception of heat and the consequent expansion be conceived as taking place in a cylinder, Figure 1, fitted with a weightless, frictionless piston, the face area of which is one square foot. Then the length of cylinder occupied by the one pound of gas is numerically equal to the volume in cubic feet, and this before heat is applied is 11.2233 feet. Atmospheric pressure at 14.7 lbs. per square inch means $144 \times 14.7 = 2116.8$ *lbs.* on the one square foot of area of the piston; and as the movement of the latter is 0.02281 lin. feet, the work done in foot-pounds is

$$2116.8 \times 0.02281 = 48.284.$$

The heat equivalent of this is

$$\frac{48.284}{777.5} = 0.0621 \ \ B.t.u.$$

This, it will be seen, is exactly the difference between the specific heat at constant pressure, 0.2173 B.t.u., and that at constant volume, 0.1552, as given in Table I. Conversely, the mechanical equivalents of the two specific heats differ by 48.284 foot-pounds; thus,

$$0.2173 \times 777.5 - 0.1552 \times 777.5 = 48.284$$
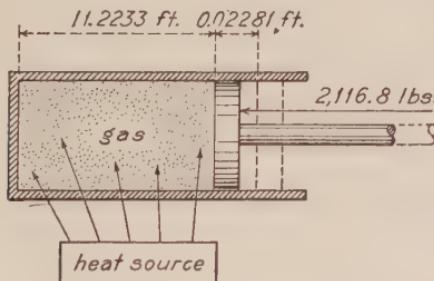
FIG. 1. HEAT ENERGY SPENT IN EXPANSION OF GAS

The quantity 48.284 as found above is in fact the constant R of page 12. Analyzed further, the computation is seen to be equivalent to

$$\frac{2116.8 \times 11.2233}{492} = 48.285,$$

and these numerical values are those which replace the symbols
in the equation,

$$\frac{PV}{T} = R;$$

hence the constant R is simply the mechanical equivalent of
the difference between the two specific-heat values of the gas.

Heat Content, Solids. — When a solid substance is heated, its
volume expands somewhat, as stated in an earlier paragraph.
The expansion is always against some resisting force, although
in many cases that force may be comparatively small, as when
a metal is heated in the open with no pressure upon it other than
that of the surrounding air. Even in that case some portion of
the heat applied is in fact transformed into mechanical energy
and effects increase of volume. In most practical cases the ex-
pansion and the pressure are so small, relatively, that the heat
equivalent of the work done in expansion amounts to only a
small fraction of a B.t.u. per pound of the substance heated.
It is so small as to be negligible in comparison with the heat
required to effect the rise in temperature. Practically all of the
heat applied, therefore, goes to increase the actual heat content,
the increase being evidenced by the rise in temperature. For
example, if a piece of brass for which the specific-heat value is
0.090 be warmed 150°, the heat content of the metal is increased
by $150 \times 0.09 = 13.5$ *B.t.u.* A very small fraction of this added
heat may have been spent, as a matter of fact, in causing en-
largement of volume, but this is not taken into account in any
ordinary computations.

Heat Content, Liquids. — When a liquid is heated, its volume
expands slightly, just as in the case of a solid. Some small
portion of the heat applied does not remain in the liquid, but
instead is used as mechanical energy to effect the volume in-
crease. While this portion is small in comparison to the total of
heat applied, it may be of enough importance to be taken into
account in certain computations. As an example, let us take the
case of one pound of liquid ammonia, heated from temperature
$- 40°$ up to 60°, at the varying saturation pressure, using values
from the U. S. Bureau of Standards Tables. (Circular No. 142.

Tables of Thermodynamic Properties of Ammonia, Department of Commerce, Washington.) The volume occupied by a pound of the liquid at − 40° temperature is given in the tables as 0.02322 cu. ft., that at 60° as 0.02597 cu. ft., the absolute pressures being 10.41 and 107.6 lbs. per square inch, respectively. Let the ammonia be imagined as being in a cylinder and supporting a weight of 144 × 10.41 = 1499 *lbs.*, as at (A) of Figure 2, the area of the cylinder section being one square foot, the apparatus being so arranged that no pressure other than that of the weight is on the surface of the liquid. At − 40° the weight is at distance 0.02322 ft. above the bottom of the cylinder and is maintained at that
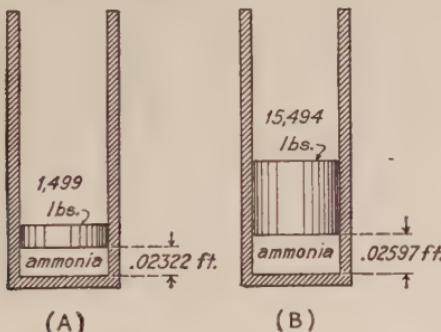
FIG. 2. HEAT ENERGY SPENT IN EXPANSION OF LIQUID

height by the liquid; hence its potential energy is 1499 × 0.02322 = 34.8 *foot-pounds.* Now let heat be applied to the liquid from any convenient source, in amount such that its temperature rises to 60°. It is necessary to increase the pressure (weight supported, in this case) in proportion, otherwise vaporization would begin; and when at 60°, the pressure must be 107.6 × 144 = 15494 *lbs.*, as at (B). Volume increases as heat is received, and the work done is that of raising the varying weight to height 0.02597 feet, the net effect being that the potential energy of the weight finally supported is 15494 × 0.02597 = 402.4 *foot-pounds.* This increased potential energy is gained at the expense of an equivalent in heat energy, amounting to

$$\frac{402.4 - 34.8}{777.5} = 0.47 \; B.t.u.$$

Now according to the Bureau of Standards Tables, the heat content of the – 40° ammonia liquid is 0.00 B.t.u., while that of the 60° liquid is 109.18 B.t.u. per pound. This 109.18 B.t.u. is the total heat used and required to raise the pound of liquid from – 40° temperature and 10.41 lbs. pressure up to 60° temperature and 107.6 lbs. pressure; but as a matter of fact, the actual gain in heat content is less than 109.18 by the 0.47 B.t.u. found as above. Moreover, the true heat content of the liquid is not 109.18 – 0.47 = 108.71 *B.t.u.*, but something less than that, because 0.47 B.t.u. is not the equivalent of the total energy which has been expended on the liquid, but only that used to effect the specified change. The total potential energy corresponding to the volume is that which would be required to change from absolute zero volume, temperature and pressure, to the final condition, being in this case, 402.4 777.5 = 0.517 *B.t.u. per pound.* The net or true heat content, therefore, is 109.18 – 0.52 = 108.66 *B.t.u. per pound.*

The total heat, that which is expended in bringing the substance up to a certain pressure and temperature, is commonly referred to as "heat content," or "total heat." It appears in tables which list the properties of such vapors as steam, ammonia, carbon dioxide, etc. As shown above, it is not the true heat content of the liquid or vapor, but is the sum of that true heat plus the energy expended in change of volume. It might have been better had it been given some other name, such as "causative heat" or words of similar import, but the term "total heat" is now deeply rooted in the literature of the subject. The true heat content as found and discussed above is called the "internal energy," or "intrinsic energy," or occasionally the "internal heat content."

Heat Content, Saturated Vapors. — Let us consider further the case of the pound of liquid ammonia as left in the cylinder at (B) of Figure 2. If the application of heat be continued, with no corresponding increase made in pressure, the added heat will cause the ammonia to change state gradually from liquid to vapor, the temperature the while remaining constant at 60° until the last drop of liquid is vaporized. According to the

Bureau of Standards Tables, the quantity of heat necessary
to effect this complete vaporization is 518.1 B.t.u., this being
the latent heat or heat of vaporization as defined on page 7.
The tables give the volume of the pound of vapor thus formed
as 2.751 cu. ft., the increase as compared with the volume
when liquid being $2.751 - 0.02597 = 2.72503$ *cu. ft.* Refer now
to Figure 3, which depicts the initial condition at (C) and the
final at (D), it being understood that the area of the piston face
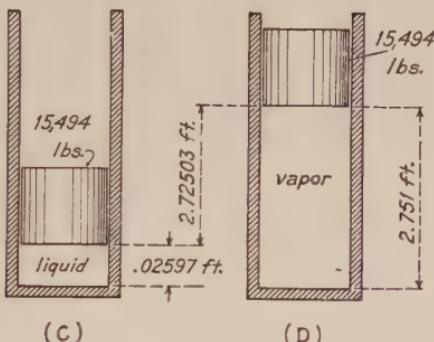
FIG. 3.   HEAT ENERGY SPENT IN VAPORIZATION

is one square foot and therefore the travel of piston in linear
feet is numerically equal to the expansion measured in cubic
feet; also that no energy is spent in overcoming frictional re-
sistance.   As the pressure is constant, the work done in the ex-
pansion is

$$15,494 \times 2.72503 = 42,221.6 \text{ *foot-pounds, or*}$$
$$\frac{42,221.6}{777.5} = 54.3 \text{ *B.t.u., heat equivalent.*}$$

The total heat required to effect the vaporization, as listed in the
ammonia tables, is 518.1 B.t.u. per pound, but of this we have
found that 54.3 B.t.u. is spent in enlargement of volume and
hence must be subtracted from 518.1 to obtain the internal
energy actually gained in the vaporizing process.   Internal en-
ergy of vaporization is listed in some of the published vapor
tables, generally with the heading " internal energy of vapori-
zation," or " internal energy of evaporation." The " total heat "

of the pound of ammonia vapor here considered is $109.2 + 518.1$ $= 627.3$ *B.t.u.*, but its internal energy content is only

$$(109.2 - 0.52) + (518.1 - 54.3) = 572.5 \text{ } B.t.u. \text{ } per \text{ } pound.$$

The internal energy of the vapor may be found by a briefer process than that followed above. It is, for any temperature-pressure condition, the difference between the total heat content and the heat equivalent of the potential energy incident to volume and pressure. For the conditions stated in our example, the volume is 2.751 cu. ft., the pressure is $107.6 \times 144 = 15,494$ *lbs.*, and the potential energy, therefore, is $15.494 \times 2.751 = 42,624$ *foot-pounds*, the heat equivalent of which is $42,624/777.5 = 54.82$ *B.t.u.* Subtracting this from the total heat we find that the internal energy is $627.3 - 54.8 = 572.5$ *B.t.u. per lb*. Obviously, the multiplier 144 and the divisor 777.5 occur in all computations of this kind, and as $\frac{144}{777.5} = 0.1852,$ the formula for finding internal energy becomes

$$Internal \text{ } energy = H - .1852 \text{ } PV.$$

In this, H = the total heat and P the pressure in pounds per square inch, absolute (all heat being per pound of vapor).

Heat Content, Wet Vapors. — Again using the example of the preceding paragraph, suppose that the vaporizing process be stopped at a point short of complete vaporization, so that while no liquid might be visible some would be suspended in the vapor in the shape of finely divided floating particles. The heat content at such stage of the process is less than that of dry, saturated vapor. If, for instance, 7 per cent of the one pound be in the liquid state, then the total heat content is

$$109.2 + (518.1 \times 0.93) = 591.0 \text{ } B.t.u.$$

Similarly, the internal energy content is

$$108.7 + (463.8 \times 0.93) = 540.0 \text{ } B.t.u.$$

Computations similar to these give the heat contents for any case.

Heat Content, Superheated Vapors. — If a pound of ammonia vapor which has been generated at 60° temperature and 107.6 lbs. pressure (absolute) be heated to some higher temperature, say to 70°, the pressure meanwhile remaining constant as for vaporization, the volume occupied by the vapor must increase. The increase, according to the Bureau of Standards Tables, will be from the 2.751 cu. ft. as at vaporization to the 2.831 cu. ft. required at 70° temperature, being 0.080 cu. ft. The total heat content as at 60° is 627.3 B.t.u. per lb., that at 70° is 634.3 B.t.u., the increase being 7 B.t.u. Now if we find what portion of this 7 B.t.u. is the equivalent of the work done in enlargement of volume and subtract it from 7, the remainder will be the gain in internal energy which accompanies the 10° rise in temperature. By a computation similar to that of page 19 for saturated vapor we have

$$\frac{0.080 \times 15{,}494}{777.5} = 1.594 \ B.t.u.,$$

which is the heat equivalent of the work done in enlargement of volume. The gain in internal energy, then, is $7.0 - 1.594 = 5.406 \ B.t.u.$ The total heat content of the pound of vapor at 70° is $627.3 + 7.0 = 634.3 \ B.t.u.$, and its internal energy is $572.5 + 5.4 = 577.9 \ B.t.u.$ The formula given in the paragraph on saturated vapors, page 20, is convenient for finding the internal energy. Using this for the example in hand, we have

$Internal \ energy = H - 0.1852 \ PV,$
$= 634.3 - 0.1852 \times 107.6 \times 2.831 = 577.9 \ B.t.u.$

The internal-energy increment found for the preceding case, 5.4 B.t.u., accompanies a 10° rise in temperature; hence the mean rate of change in internal energy is 0.54 B.t.u. per degree, this being the specific heat at constant volume for the particular pressure and temperature conditions stated. It is the heat which causes the temperature to rise. The 7.0 B.t.u., on the other hand, is the heat which causes a 10° temperature rise plus that which causes an increase in volume; it is the gain in internal energy plus the work done. It is at the mean rate

of 0.70 B.t.u. per degree, the specific heat at constant pressure. (See page 13.) The ratio of these two values, 0.70 0.54 = 1.2963, is the mean value of y for the temperature range and pressure considered. It may be noted that it differs but little from the nominal value 1.30 given in Table I for atmospheric pressure and 32° temperature, although the values for C_p and C_v are very different. All three of these quantities are variables for any vapor.

Tables giving the properties of vapors show heat content as compared with zero heat at some arbitrarily chosen temperature. For steam tables the freezing temperature, 32°, is the base or datum commonly used, the total heat content at that temperature being called 0.00. Tables of properties of ammonia, carbon dioxide, etc., likewise may have 32° as the datum, but these are used as refrigerants at temperatures as low as $-30°$ or $-40°$; and if 32° be the datum, any computation of heat content for a temperature lower than 32° must involve negative quantities. Hence it is somewhat more convenient to use a low temperature datum, such as $-40°$, on which the Bureau of Standards Ammonia Tables are based. Of course, the substance has a certain heat content at the basing temperature, or at any temperature above absolute zero; but it is neither practicable nor necessary to determine what it is, the user of the tables being concerned only with changes in heat content which take place between possible working pressures and temperatures.

Heat Content, Ideal Gases. — When working with the so-called ideal gases, such as air, oxygen, etc., we have neither the heat of a liquid nor heat of vaporization to be taken into account, and the values for C_p and C_v are so nearly constant that they may be considered such for most practical purposes. As the behavior of these gases follows closely the laws stated on pages 12-15, it is an easy problem to compute volumes, temperatures, pressures, total heat, and internal energy when the initial conditions are known. For example, if at constant pressure the specific heat of air be 0.2396 (Table I), and at constant volume 0.1711, then to find the total heat content

and the internal energy of a pound of air at any temperature it is only necessary to multiply each specific-heat constant by the difference between the specified temperature and that selected as a basis for reference. The latter may be any which is convenient, such as 32°, or zero, or − 100°, etc. Thus if − 100° be the reference datum, the total heat content of a pound of air at 90° temperature is

$$190 \times 0.2396 = 45.5 \ B.t.u.$$

Similarly, the internal energy at same temperature is

$$190 \times 0.1711 = 32.5 \ B.t.u.$$

The difference between these two heat contents, 13.0 B.t.u., is the heat equivalent of the mechanical energy required to increase the volume of the one pound of air from whatever it may be when at − 100° temperature to that which it becomes at 90° temperature, and this regardless of what the initial, intermediate, and final pressures may be. It is apparent that as the value of the ratio y is in this case constant, 1.400, the internal energy may be found simply by dividing the total heat content by 1.400; thus, 45.5/1.4 = 32.5. Conversely, the internal energy multiplied by the value for y gives the corresponding total heat.

External Work. — In preceding paragraphs it has been shown how in the heating of a liquid, in the vaporization of the liquid, and in the superheating of the vapor, some portion of the heat applied is transformed into mechanical energy and as such is expended in the enlargement of volume against external resistance. The latter may be only the pressure of the atmosphere on the rising vapor or on the liquid being warmed, or it may be that from a piston, an imposed weight, etc. Whatever be the form of the resistance, work is done against it, the work being the mechanical equivalent of a definite portion of the total heat used. It is the "external work" done during the heating process. The energy resultant from expenditure of heat in doing external work may be left in available form or

may be wasted in so far as the gas or vapor is concerned. In the case illustrated by Figure 3, for example, the raised weight may or may not be made to serve some useful purpose.

When a vapor is to be condensed to liquid at a constant pressure, external work must be done upon it by some outside force to reduce its volume from that required for the vapor to that which suffices for the liquid. This external work done in condensation is exactly equal in amount to the external work accomplished by heating during the formation of the vapor. To illustrate, let the vapor-filled cylinder shown at (D) of Figure 3 be placed in horizontal position, as at (E) of Figure 4, and let some cold substance be arranged to serve as a heat receiver (cold water, for instance, as in an ordinary condenser) while the pressure is held constant at 15.494 lbs. on the one square foot area of the piston face. Heat then flows to the cold substance from the ammonia, but the temperature of the latter does not fall as long as any uncondensed vapor remains in the cylinder. Meanwhile the piston must be gradually pushed in by some force acting on the piston from without. The heat flowing to the heat receptor comes from two distinct sources: first, the ammonia parts with its "internal energy of vaporization," its true latent heat which it had retained while existing as vapor, amounting in this case to 463.8 B.t.u. per lb.; and second, the work done by an external force pushing in the piston while the ammonia volume shrinks is transformed into heat, and this also passes to the heat receptor. The second item amounts to 54.3 B.t.u. in this case (as found on page 19). The sum of these two heat items is the total heat of vaporization at the temperature and pressure stated.

It may be well to emphasize here that vaporization of a liquid at a constant pressure cannot be effected without expenditure of a certain quantity of heat in doing external work; nor when the process is reversed can condensation be effected without expenditure of mechanical energy by some force acting from without, this energy in turn being transformed into heat. Referring again to (C) of Figure 3, if the weight or piston there shown were to be fixed in position so that it could not rise

and heat then were imparted to the liquid ammonia as before, no work could be done; no part of the heat applied would be transformed into mechanical energy and expended as such. The case would be that of heating with volume held constant, and the pressure would rise in proportion to the rise in absolute temperature, in accordance (approximate) with Charles's law. For the reverse case, subtraction of heat, suppose that the piston be securely fixed as in the position shown at (E) of Figure 4 and that cold water or some other heat receptor then be applied as indicated. Again, no work could be done, as no force acting outside of the cylinder would be expended as mechanical energy; and whatever cooling might take place
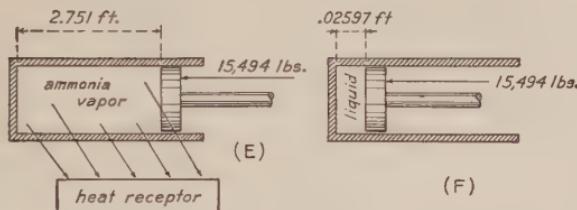
FIG. 4. EXTERNAL WORK REQUIRED IN CONDENSATION

would be with constant volume of the vapor, which would gradually cool until its temperature should be equal or very nearly equal to that of the heat receptor, the cooling being accompanied by a gradual drop in the vapor pressure.

Entropy. — Entropy may be thought of as the ratio between the total heat-change necessary to effect a certain change in a substance and the absolute temperature of the substance during that change. The change may be in temperature, pressure, state, or (and) volume. We shall consider, first, a change in state, as when a liquid is vaporized, and for our purpose the values used for the example on page 19 are convenient, since it has been shown there that the total heat required to vaporize a pound of ammonia at 60° temperature is 518.1 B.t.u. The absolute temperature, which is $60 + 459.6 = 519.6°$, remains constant throughout the change of state. The entropy

value for the change is the ratio of the total heat used to the absolute temperature, thus,

$$Entropy = \frac{518.1}{519.6} = 0.9971.$$

This is the " entropy of vaporization."

Any liquid (or solid) at any temperature has a certain intrinsic entropy, its value being dependent on all changes in temperature, pressure, and state which may have occurred in raising its temperature from absolute zero. It is not possible to compute this entropy value, on account of our lack of knowledge of the specific heats, heat of fusion, etc., for low temperatures. For practical purposes we are concerned only with changes in entropy, the intrinsic or absolute entropy being of little or no interest; hence it is sufficient to call the entropy 0.00 at some arbitrarily chosen temperature. In the construction of tables of properties of vapors, such as those for steam, ammonia, sulphur dioxide, etc., it is customary to take for the entropy base the same temperature as that at which the heat content is called 0.00. In the Bureau of Standards Ammonia Tables, that temperature is − 40°; in Marks and Davis's Steam Tables it is 32°; in some other tables, 0°, etc.

If a pound of water be heated through the one degree from 32° to 33°, it requires the expenditure of one B.t.u. of heat. (Page 2.) The temperature at which this is done varies gradually from 32° to 33°, the average being 32.5°, which is 32.5 + 459.6 = 492.1° *absolute*. The ratio of heat used to average absolute temperature is therefore $\frac{1}{492.1} = 0.00203$, this being the *increase* in entropy for the pound of water when heated from 32° up to 33° temperature. If, now, its entropy at 32° arbitrarily be called zero, at 33° its entropy is 0.00203. Similarly, if further heated from 33° to 34°, the entropy change for this degree is $\frac{1}{493.1} = 0.00203$; making its total entropy, reckoned above the chosen zero base

$$0.00203 + 0.00203 = 0.00406.$$

This is "entropy of the liquid," a column with this heading being found in all tables giving properties of vapors. The approximate method of computation as here given, using average or mean absolute temperature, does not cover the cases where there is a change in pressure nor is it sufficiently accurate for the construction of working tables, but it may serve to show the principle involved.

The entropy of a saturated vapor is the sum of the entropy of the liquid from which it was generated and its entropy of vaporization. For example, when ammonia boils at 20 lbs. pressure (absolute, per square inch), the entropy of the liquid at the corresponding temperature, $-16.6°$, is 0.0578; the entropy of vaporization is 1.3122; and the sum of these two items, 1.3700, is the "entropy of the vapor."

The entropy of a superheated vapor is the sum of three component parts: entropy of the liquid, entropy of vaporization, and entropy of superheat, the last being the ratio of the heat expended in superheating to the absolute temperature. To illustrate, the entropy of saturated steam at 60 lbs. absolute pressure is 1.6432, according to Marks and Davis's Tables. If a pound of this steam be superheated through 20 degrees, which means from $292.7°$ up to $312.7°$, the total heat increment is found from the tables to be 10.3 B.t.u. The increase in entropy, or entropy of superheating, is approximately $\frac{10.3}{762.3} = 0.0135$. (The average absolute temperature is $762.3°$.) Adding this to the entropy as for saturated steam, we have $1.6432 + 0.0135 = 1.6567$, the entropy of the superheated steam. The value given by Marks and Davis is 1.6568. It is, of course, not strictly correct to use the average absolute temperature as a divisor, since it is not the true mean temperature, but in the construction of tables for actual use the utmost refinement in method is necessary.

The entropy of a wet vapor, per unit weight, is the sum of the entropy of the liquid plus the entropy of vaporization of that portion which is vapor. For example, if at a certain pressure the entropy of vaporization for saturated steam be 1.2160, and

the entropy of the liquid be 0.4272, then the entropy for steam of 95 per cent quality is 0.4272 + (1.216 × 0.95) = 1.5824.

In dealing with wet vapors, it is sometimes necessary to find what quality will be required for a vapor to have a certain prescribed entropy. The process is the reverse of that just given, and is conveniently expressed as a formula, thus:

$$Required\ Quality = \frac{T_s - L_s}{V_s}.$$

In this formula

T_s = *total entropy of wet vapor for which the quality is sought,*
L_s = *entropy of the liquid, and*
V_s = *entropy of vaporization; that is, amount per pound.*

As an example in the use of this formula, let it be required to learn what quality of steam at 60 lbs. absolute pressure will have entropy 1.5824. The entropy of the liquid (found in the tables) is 0.4272, and that of vaporization is 1.2160. Substituting these in the formula, we have

$$Required\ Quality = \frac{1.5824 - 0.4272}{1.2160} = 0.95.$$

Finally, we have to consider the entropy of perfect gases. Here there is neither entropy of liquid nor of vaporization to be taken into account. If such a gas be heated without change of pressure, the entropy change is represented by the ratio of the total heat expended to the absolute temperature during the heating. The latter, of course, rises gradually as heat is applied. The use of the mean or average absolute temperature will give results which are only approximately correct. To find the exact change in entropy, letting T_1 represent the lower and T_2 the higher temperature, the following formula applies:

$$Entropy\ change = C_p \times 2.3026 \times \log\left(\frac{T_2}{T_1}\right).$$

As an example in the use of this formula, let air be heated from 60° to 200° temperature, at any constant pressure. The absolute temperatures being 520° and 660°, and the specific heat, C_p, being taken as 0.24, we substitute these values in the formula, and have

Entropy change = 0.24 × 2.3026 × 0.1035 = 0.0562.

Should one desire to use the approximate method for finding this change, the first step is to find the total heat expended, which is (200 − 60) × 0.24 = 33.6 *B.t.u.* The average absolute temperature being 590°, the ratio of the total heat change thereto is

$$33.6 \ / \ 590 = 0.0569.$$

In most practical problems there is a change in pressure as well as in temperature and volume. If, however, there be change in pressure and temperature without change in volume, the change in entropy is expressed by the following formula:

$$Entropy \ change = C_v \times 2.3026 \times \log\left(\frac{T_2}{T_1}\right).$$

If we have change in temperature, volume, and pressure, the general case, the entropy change is thus expressed in formula:

Entropy change

$$= C_p \times 2.3026 \times \log\left(\frac{T_2}{T_1}\right) - \frac{R}{777.5} \times 2.3026 \times \log\left(\frac{P_2}{P_1}\right).$$

In this P_1 is the low pressure and P_2 the higher pressure. The constant R is that explained on page 12.

CHAPTER II

HEAT CHARTS

Heat-Temperature-Entropy Chart. — Charts can be made to show graphically the same relations among heat content, temperature, volume, and entropy of a vapor which appear in tables giving the properties of that vapor. These are known as " heat charts " or as " Mollier charts, " the latter in honor of the scientist to whom we are indebted for their development. (Mollier, Germany.)

Figure 5 is one form of the Mollier chart, showing the properties of ammonia, being drawn from data embodied in the U. S. Bureau of Standards Circular No. 142, *Tables of Thermodynamic Properties of Ammonia* (by permission of the Bureau). Volume curves have been omitted in order to simplify the construction. A large chart more extended in scope than Figure 5 and comprising curves for volume is contained in the publication named. Charts of this general character but perhaps differing in arrangement are to be found in other published works on properties of vapors.

Total Heat Chart, Ammonia. — Reference to Figure 5 shows that at any point on the saturation curve the absolute pressure, ordinary temperature, total heat content, and entropy are determined by the several curves which intersect at that point. After some practice one can take readings from such a chart with a degree of accuracy sufficient for many purposes; but when extreme accuracy is required, it is necessary to make use of the tables instead. That part of the chart which is to the right of the saturation curve applies to superheated vapor, while that to the left applies to wet vapor. The Bureau of Standards chart is extended in this direction to cover the entire range of quality and the liquid line, thus making it more serviceable for problems in refrigeration.
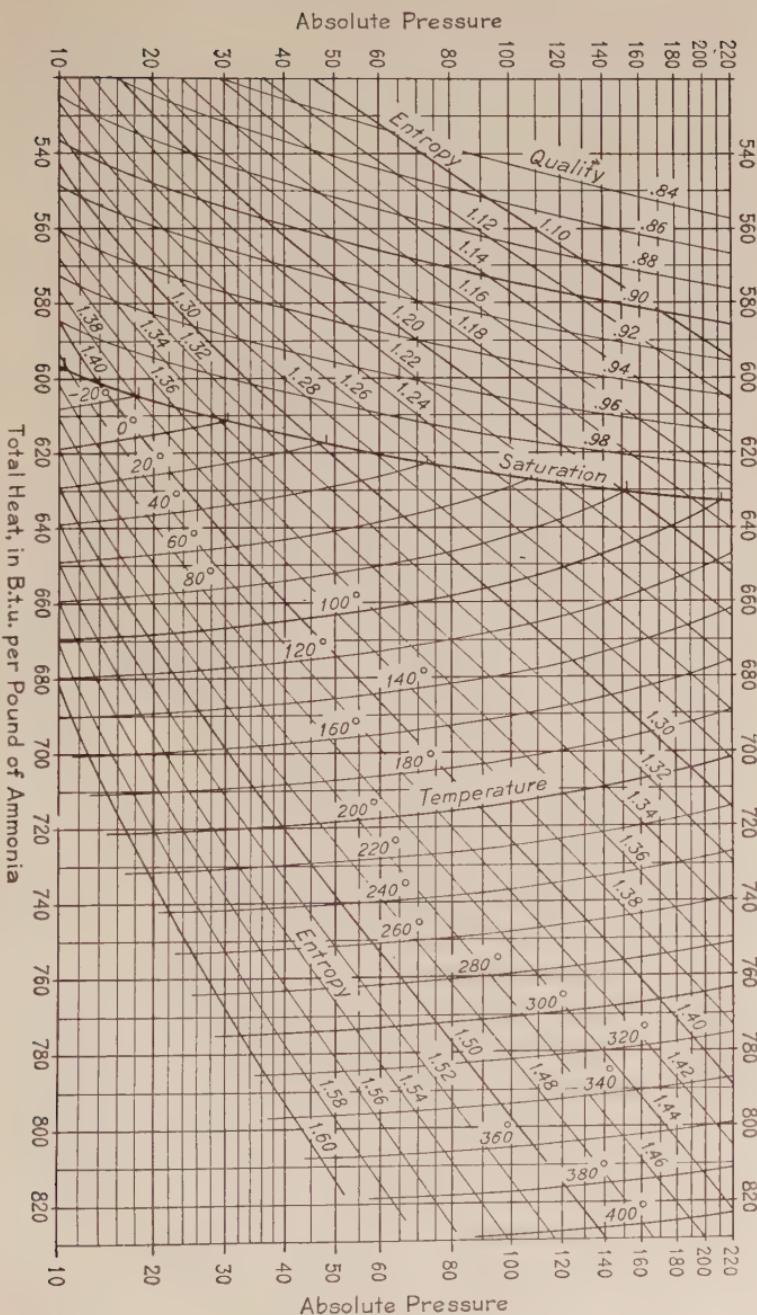
FIG. 5. TOTAL HEAT CHART FOR AMMONIA VAPOR

A few examples in chart reading will be given. At 100 lbs. pressure the corresponding saturation temperature is seen to be about 56°, the total heat content 626.5 B.t.u. per pound, and the entropy about 1.236. Passing to the right along the line for 100 lbs. pressure (all pressures in this and the other heat charts are absolute pressures) until the curve for 200° temperature is met, one sees that the total heat content as at that temperature is 714.0 B.t.u. and the entropy 1.385. To find the temperature which vapor at 20 lbs. pressure should reach in order to have this same entropy, follow the line for entropy 1.385 (imaginary line, in this case, but readily traced on the chart) down to its intersection with the horizontal line for 20 lbs. and read the required temperature as − 4° or − 5° and heat content as 613.0 B.t.u. Continuing along the line for entropy 1.385 to pressure 10 lbs., we cross the saturation curve and find that the quality of vapor having that entropy is 97.1 per cent and that its heat content is 579.0 B.t.u. per pound. Thus we may read the properties for any condition within the scope of the chart, for wet, saturated, or superheated vapor.

Internal Energy Chart, Ammonia. — A chart having the same arrangement of curves as Figure 5 but using internal energy instead of total heat is presented as Figure 6 of this work. It is used in the same manner as Figure 5. Readings for temperature and entropy from the two charts should be practically identical for any case. Thus, for 100 lbs. pressure the saturation temperature is 56° and the entropy 1.236, same as read from Figure 5. The internal energy is read as 572.0 B.t.u. per pound. If the 100-lb. vapor be heated to 200° temperature, its internal energy becomes 639.0 B.t.u., its entropy being 1.385 as already found in Figure 5. The basing data for this chart are the volumes and total heat content listed in the Bureau of Standards Tables, the internal energy having been computed from these by the writer. (See page 20.)

Total Heat Chart, Steam. — Figure 7 is a heat chart for the vapor of water, steam, similar in construction to Figure 5 for ammonia. Data for construction of this chart have been

FIG. 6. INTERNAL ENERGY CHART FOR AMMONIA VAPOR
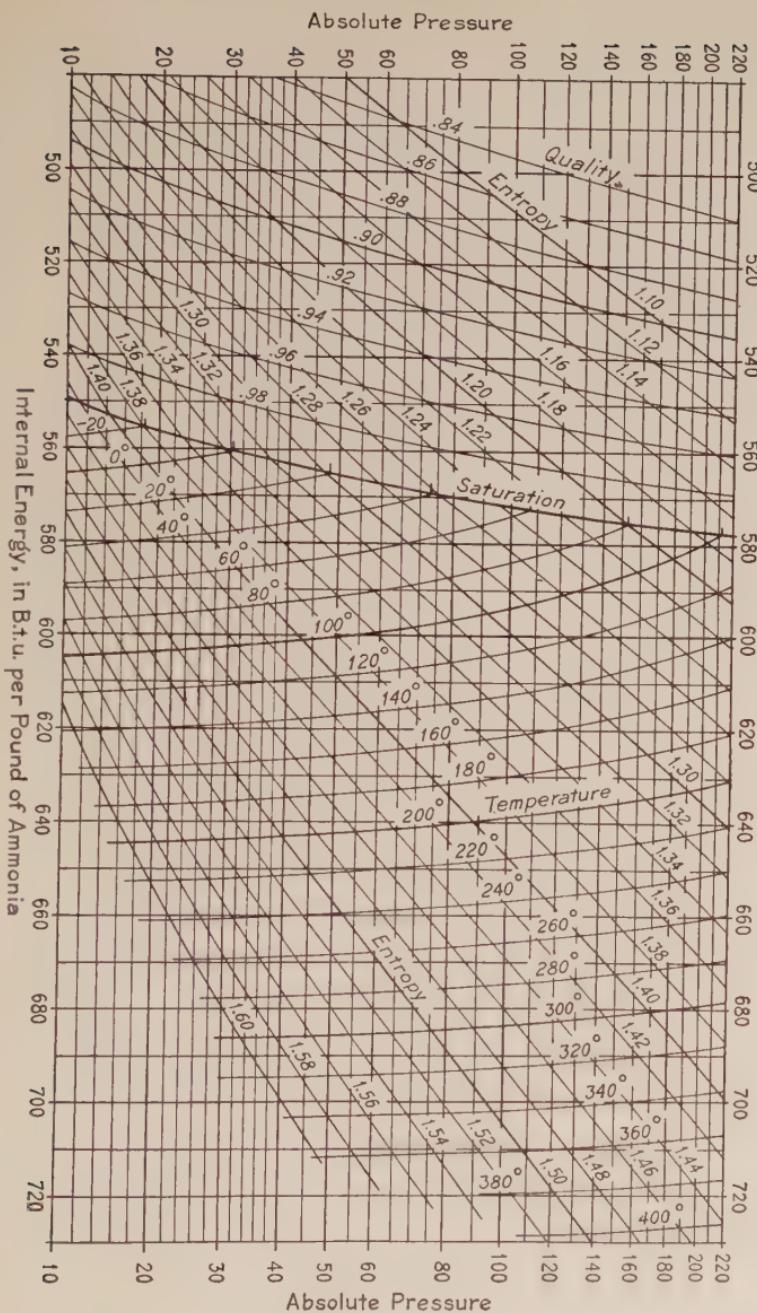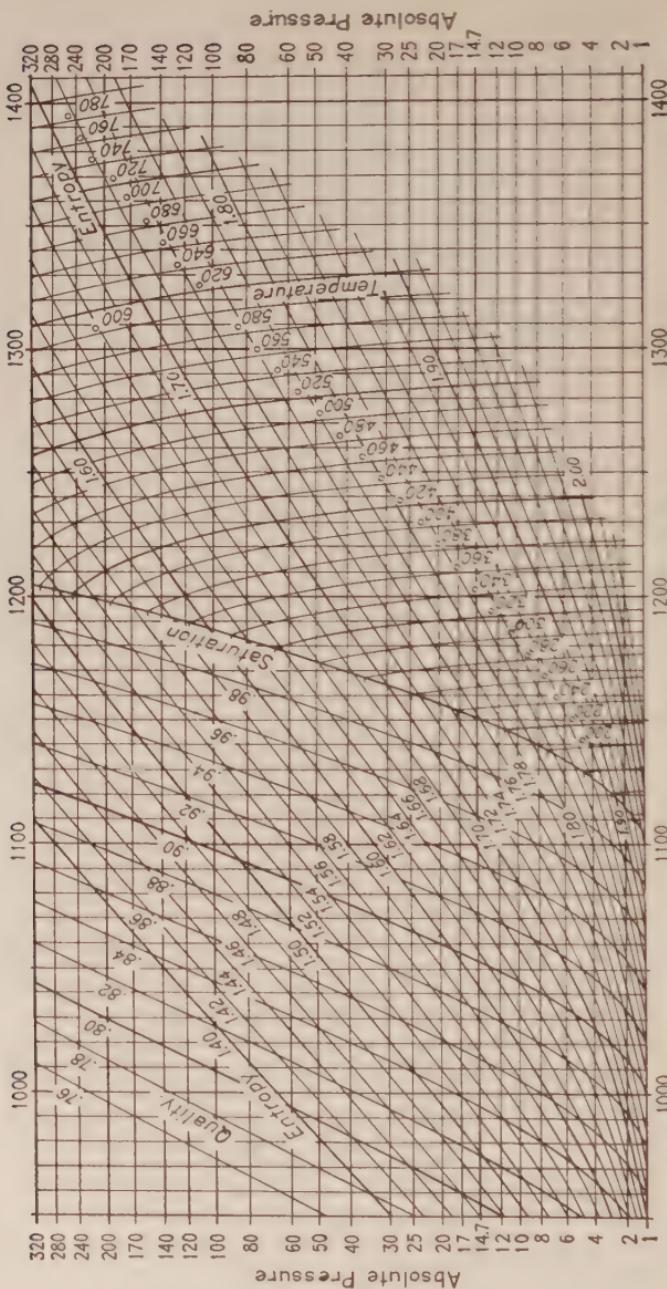
FIG. 7. TOTAL HEAT CHART FOR STEAM
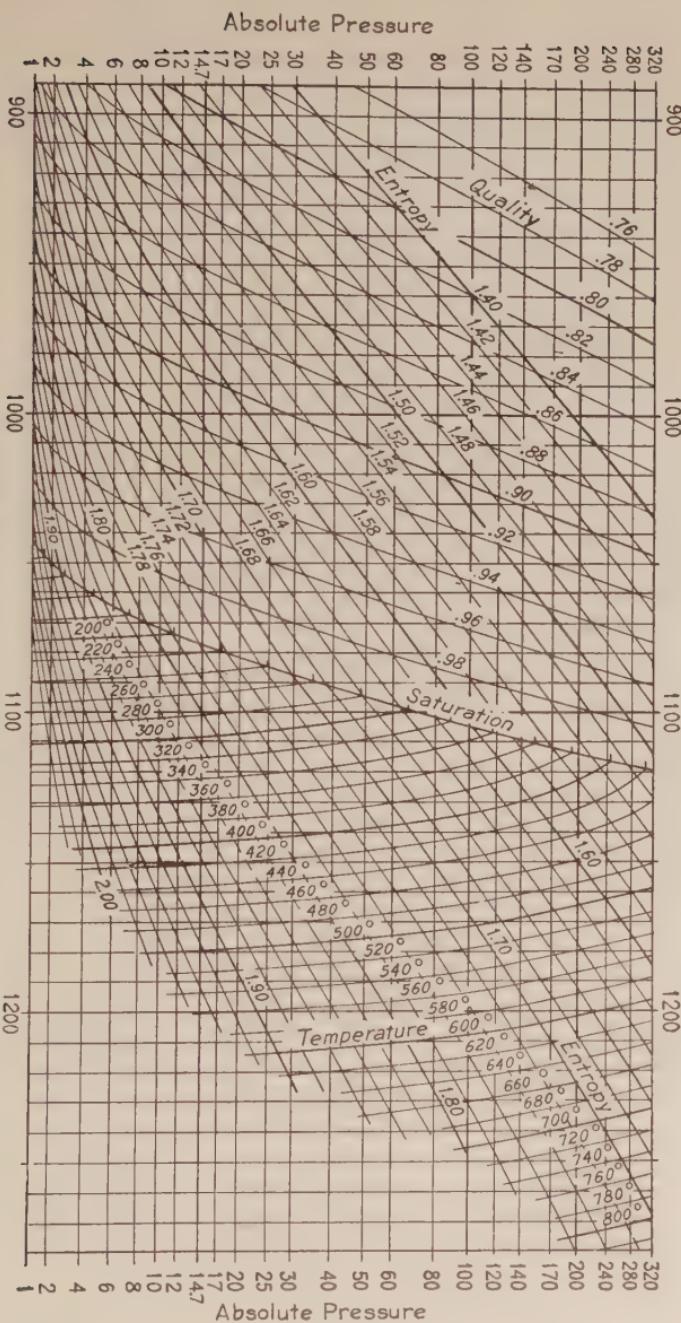
FIG. 8. INTERNAL ENERGY CHART FOR STEAM

taken from Marks and Davis's Steam Tables, by permission of the publishers. The arrangement of this chart differs from those which accompany the Marks and Davis publication, having been made to conform to that used in Figures 5 and 6. Volume curves are likewise omitted. The heat content shown is total heat.

Internal Energy Chart, Steam. — Figure 8 is a second Mollier chart for steam, plotting internal energy content instead of the total heat. Readings of temperature and entropy should agree with readings from Figure 7. The basing data are from the tables of Marks and Davis; the computations for internal energy and those made necessary by the change in arrangement having been made by the writer.

Heat Chart, Air. — It would be possible to construct a chart for air with curves for entropy, temperature, pressure and heat content, and for volumes as well. The heat content might be the total heat, as in Figures 5 and 7 or the internal energy, as in Figures 6 and 8. Such charts (for total heat, at least) have been published and are available to those interested. For the purposes of this work the special chart shown as Figure 18 has been devised. In that chart the horizontal lines represent ratios of pressure instead of pressures. Heat content is independent of pressure, as is the case with any ideal gas, being a function of the temperature alone and easily read from the scale along the top of the chart. Thus the one chart shows both the total heat and the internal energy content. The oblique lines are in fact lines for entropy, but the value for any line varies with pressure and cannot be written on the chart. The graduations for the vertical scale (pressure ratios) have been varied in such manner that the entropy lines plot straight; with a normal scale, they would plot as curves. The uses of this chart are discussed in later paragraphs (pages 71, 73).

CHAPTER III

COMPRESSION OF GASES AND VAPORS

Adiabatic Compression. — If a certain weight of a gas or vapor be placed in a cylinder and compressed by means of a piston (which is assumed as working against the cylinder walls without friction), the mechanical energy expended in compression is transformed into heat, which in turn passes into the gas being compressed. This heat may or may not remain in the gas. If all of it remains and if the gas receives no heat from other sources, then the compression is "adiabatic," that word meaning, in substance, "no heat passing through." This condition is seldom or never attained in actual work, as there is practically always more or less heating by friction of the piston against the cylinder metal and always some loss of heat by radiation and conduction.

Isothermal Compression. — It is possible in theory and sometimes in actual practice to carry away all of the heat of compression, so that the temperature of the gas after compression to small volume is precisely the same as the temperature at which the compression stroke begins. This is known as "isothermal" compression, which means, substantially, "with equal heat." As commonly used, the word "isothermal" applies to the condition of equal temperatures rather than to that of equal heat contents. The distinction in meaning is of no importance when dealing with the ideal gases, as the heat contents of an ideal gas are equal at equal temperatures, regardless of pressures, but this is not true of vapors; the heat content of a vapor varies with pressure as well as with temperature.

If isothermal compression is to be maintained, the heat of compression must be carried away as fast as it is generated. To effect this the compressor cylinder is provided with a water-jacket through which cold water is circulated, the water acting

as a heat receptor. Heat flows by conduction through the cylinder metal and from that to the cooling water, and in addition a certain amount of heat is radiated to the surrounding atmosphere.

Polytropic Compression. — Any compression (or expansion) of a gas or vapor which is neither adiabatic nor isothermal is said to be "polytropic," that word meaning, in substance, "having many forms." Polytropic compression obtains when
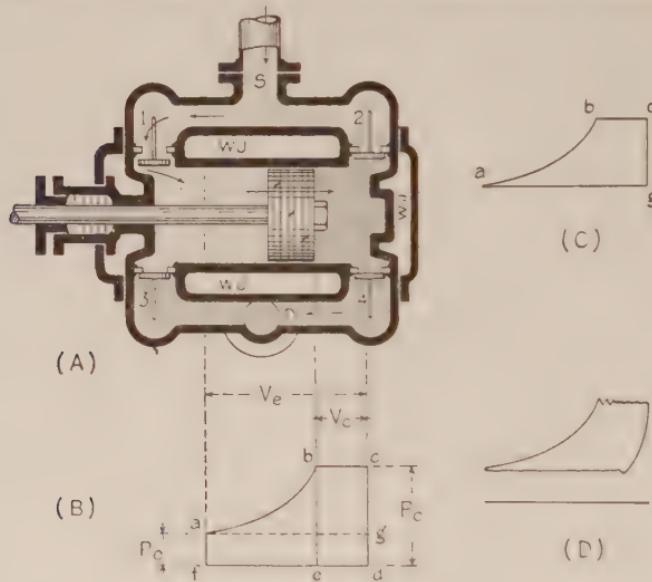
FIG. 9.   TYPICAL COMPRESSOR CYLINDER AND PRESSURE-
VOLUME DIAGRAMS

any heat either more or less than the heat of compression is imparted to an outside substance, such as cylinder metal, air, water, the cold gas entering the cylinder, etc. One may generalize and say that practically all compression or expansion of gases or vapors taking place in commercial apparatus, whether in air or gas compressors, pneumatic or steam engines, pneumatic tools, automotive engines, etc., is polytropic in character.

The Mechanics of Compression. — The essential element of the ordinary gas or air compressor is a cylinder (one or more) as sketched at (A) of Figure 9, with a working piston which is

made to fit as tightly as may be without excessive friction. The machine may be "single-acting," compressing gas during the outbound stroke only, or it may be "double-acting," compressing during both the outbound and the inbound strokes of the piston, the latter being the type sketched in Figure 9. The inlet or suction valves are marked 1 and 2, the discharge valves 3 and 4. The incoming low-pressure gas enters through the suction or inlet main, S, and the compressed gas is forced out into discharge pipe, D. A chamber or water-jacket, WJ, is usually provided around the cylinder barrel and in some machines over the cylinder heads as well, cool water being circulated by pump through this chamber to take up some part or all of the heat of compression.

Starting with the cylinder full of low-pressure vapor or gas, the moving piston compresses it from its original volume, indicated by length V_e, to some reduced volume indicated by length V_c, the piston face moving from a–f to b–e. During this part of the stroke the discharge valve, 4, remains closed, the discharge pipe already being full of gas at the same pressure as that inside the cylinder at the instant volume V_c is attained. At that instant valve 4 should open, although in practice the inside pressure must be enough higher than that in the discharge manifold to force open the valve, which if properly set and constructed offers little resistance. The piston, continuing its travel to the end of stroke, pushes the compressed gas out into the discharge main without further altering its volume or increasing its heat content. Heat generated in compression is imparted to the gas during the compressive part of the stroke. If all of the heat of compression be thus imparted to the gas during this portion of the stroke, none of it passing to jacket water, metal, the air in the engine room, or to the cool incoming gas; and if no heat be received by the gas from piston friction or other outside source, so that the gas carries its full burden of heat of compression, and no more, at the instant compression ceases, then the compression is adiabatic.

If the jacket water, air, metal, incoming cold gas, etc., take up all of the heat put into the gas by compression, and no

more, so that temperature and heat content of the gas at the
instant when compression ceases at b–e are the same as when
it begins at a–f, then the compression is isothermal (referring
to the case of an ideal gas). If some of the heat of compression
remains in the gas as compression ceases, so that its temperature
and heat content are greater than at the beginning, the com-
pression is polytropic. If some of the heat of compression
passes temporarily to the piston or cylinder metal during the
compressive part of the stroke, and from that metal to the
compressed gas while it is being pushed out into the discharge
main by the expulsive part of the stroke from b–e to c–d, the
compression is polytropic, even though the temperature of
the gas entering the discharge main may be exactly that of the
gas as at beginning of compression. Finally, if the cool gas
entering the cylinder be warmed to any degree before compres-
sion begins by heat resident in the cylinder or piston metal,
but has the same temperature at b–e that it had before enter-
ing the cylinder, again the compression is polytropic; final
temperature is lower than initial temperature.

Work Done in the Compressor. — Work done is equal to
the product of the pressure or force acting multiplied by the
distance through which it acts. In the course of the compres-
sive part of the piston stroke, the pressure varies from P_c,
that after passing the inlet valves, to P_c, that inside the cylin-
der at the instant the discharge valves unseat. See sketch
(B) of Figure 9. The work done is represented graphically by
the area under the compression curve from a to b, that is, by
area a–b–e–f, which represents the product of varying pressure
multiplied by distance moved. When the discharge valve
opens, compression ceases; and the work for the remainder of
the piston stroke is that of expelling the compressed gas. No
heat is generated in this part of the stroke other than that due
to friction of the moving parts. If there were no frictional
heat and no delayed heat transfer from metal to gas, the
temperature, heat content, and volume per unit weight of gas
would be the same at the discharge valve as at the point where
compression of volume ceased.

The work done in expulsion of the compressed gas is the product of the final pressure multiplied by the final volume or $P_c \times V_c$. If the pressure be stated in pounds per square foot, and length V_c in feet, then the product, which is work done, is in foot-pounds. On the diagram it is represented by the rectangular area b–c–d–e. The sum of the work of compression and the work of expulsion is the total work done against or on the gas through one piston stroke.

Throughout the compression and expulsion the low-pressure gas entering the cylinder through the inlet valve is pressing against the rear face of the piston and thus doing a certain share of the work. This contra-work by incoming gas is equal to the product of the low pressure multiplied by the length of stroke; that is, it is $P_e \times V_e$, and the work is represented by area a–g–d–f of the diagram. The net work required of the motive power, then, is represented by area a–b–c–g, the sum of the compression and expulsion work minus the contra-work by incoming gas. It is what is shown by an ordinary indicator diagram, as sketched at (D) of Figure 9, except that the indicator diagram shows the effects of clearance and other minor factors, as discussed in a later chapter.

The work of expulsion of the compressed gas is not altered by any gain or loss of heat to or from the gas while it is being expelled, provided there be no change in discharge pressure. The distance traversed by the piston during gas expulsion, from b–e to c–d, is not affected by any change in the temperature, heat content, or volume per unit weight of the gas being expelled.

If the compression be isothermal, the product of final volume multiplied by final pressure is equal to the product of initial volume multiplied by initial pressure. That is

$$P_c V_c = P_e V_e.$$

This being true, it follows that area b–c–d–e, Figure 9, is equal to area a–g–d–f, and hence true that the total work, net, is the same as that of the compression part of the stroke. In the ordinary working of a compressor, where the discharge tem-

perature is somewhere between the isothermal and the adiabatic, the work of expulsion exceeds the contra-work. That is, in this case

$$P_c V_c \text{ is greater than } P_e V_e.$$

With favoring conditions it might be possible to carry away so much heat by jacket water and radiation that the temperature of the gas after compression would be lower than at beginning of the stroke, in which unusual case the work of expulsion would be less than the contra-work done by the incoming gas; that is

$$P_c V_c \text{ would be less than } P_e V_e.$$

Work of Isothermal Compression. — It has been shown that the work done in compression is represented by the area under the compression curve, and that in the special case of isothermal compression of a perfect gas, this work is equivalent to the net total work for the complete piston stroke, neglecting for the present the effects of clearance and other minor factors. It is demonstrated in complete treatises on thermodynamics that this work of isothermal compression, in foot-pounds per pound of gas compressed, is given by the following formula:

$$\textit{Work of isothermal compression} = P_c V_c 2.3026 \log \left(\frac{P_c}{P_e} \right). \tag{1}$$

It will be noted that, as $P_c V_c = P_e V_e$, the latter values may be used in place of the former if more convenient, as would be the case when finding the work done by a gas expanding isothermally, for which this formula is equally true. Also, as the volumes are in inverse ratio to the pressures, that is, $\dfrac{P_c}{P_e} = \dfrac{V_e}{V_c}$, the volume ratio as thus written may be substituted for the pressure ratio appearing in the formula. The pressures used in the formula are in pounds per square foot, absolute. If pressures per square inch be used, the result must be multiplied by 144 to convert it from inch-pounds to foot-pounds. (The pressure ratio, of course, is the same whether the pressures be per square foot or inch.) The work in foot-pounds is con-

vertible into heat equivalent by dividing by 777.5, the work equivalent of one B.t.u.

It has been shown (page 13) that $PV = WRT$; hence when the weight of the gas, W, the gas constant R, and the absolute temperature T are known, the product of these may be used in place of $P_e V_e$ in formula (1), thus making it unnecessary to find the gas volume. With this substitution, and for one pound weight of gas, the formula becomes:

$$Work\ of\ isothermal\ compression = RT\ 2.3026\ \log\left(\frac{P_c}{P_e}\right). \quad (2)$$

This formula gives the work in foot-pounds, multiplication by 144 not being required. Division by 777.5 converts to B.t.u. of heat equivalent.

In the last formula for entropy on page 29, the subtractive term of the right-hand member of the equation represents the change in entropy due to change of pressure without change of temperature. Comparison of this with formula (2) shows that when multiplied by the absolute temperature it is equivalent to what the right-hand member of (2) becomes when divided by 777.5 to convert to B.t.u. Therefore, for an isothermal compression or expansion of a perfect gas, the work done, in B.t.u. heat equivalent, is equal to the change in entropy multiplied by the absolute temperature at which the compression or expansion is done.

As an example in finding the work done in isothermal compression, let us take a pound of oxygen gas, with initial pressure 20 pounds per square inch, absolute, and compress it to 90 lbs., the initial and final temperature being 70°. This temperature, be it noted, is that of the oxygen at beginning of compressive stroke and at the instant the discharge valve opens. The temperature of the gas before passing the inlet valve may be less (or more) than 70° and that of the discharged gas may be more or less. The pressures are known and the value of the constant R is to be had from Table I; hence formula (2) may be used. Substituting in it the several proper values, we have for one pound weight of oxygen

Work of compression $= 48.283 \times 529.6 \times 2.3026 \times log.$ *of* 4.5,
$$= 38,460 \ foot\text{-}pounds = 49.47 \ B.t.u.$$

If the horse-power rate be required we must know the time necessary for the compression. If one pound be compressed per second, the power expended directly on the gas is

$$\frac{38,460}{550} = 69.92 \ H.P.; \ \text{or, using the heat equivalent, it is,}$$

$$\frac{49.47 \times 60}{42.45} = 69.92 \ H.P.$$

It will be noted that this does not include any work done in overcoming frictional resistance. There is always more or less friction, requiring expenditure of additional power, and its heat equivalent must be carried away by the jacket water, air, metal, etc., if the compression is to be isothermal.

If a table or a Mollier chart were to be made for oxygen gas, showing the entropy at practicable temperatures and pressures, the difference between the entropies shown therein for temperature 70° and pressures 20 and 90 lbs. would be found to be 0.09342. This change in entropy multiplied by the absolute temperature gives a result identical with that obtained by use of formula (1) or (2), thus,

$$0.09342 \times 529.6 = 49.47 \ B.t.u.$$

Obviously, if such a table or chart be available, this is the simpler method for finding work of isothermal compression.

Isothermal Compression of Vapors. — It has been stated (page 9) that Boyle's law is only approximately true for vapors. The specific heat of a vapor is a variable quantity; hence vapor volumes as at a given pressure do not vary directly with the absolute temperature as in the case with an ideal gas, and the product of pressure multiplied by volume is not constant at a given temperature. It will be found that, in general, the product of PV is more nearly constant for the condition of equal heat content than for the condition of equal temperature. Taking an example from the ammonia tables, at 35 lbs. pressure and 70° temperature, the volume per lb. of vapor is 9.289 cu. ft., the

PV product, therefore, being $35 \times 9.289 = 325.1$. Now choosing some other pressure at random, say 115 lbs., at 70° the volume per lb. is found to be 2.628 cu. ft. and the PV product is $115 \times 2.628 = 302.2$, this differing considerably from that first found. Now if we find the temperature at which the total heat content of the 115-lb. vapor equals that of 35-lb., 70° vapor, it being 96°, we shall find that the volume per lb. at that temperature is 2.813 cu. ft., and the PV product 323.5, this differing but a little from that for the 35-lb. vapor.

Similarly, using internal energy instead of total heat, we find that the temperature of the 115-lb. vapor having internal energy equal to that of the 35-lb., 70° vapor, is 95.4°, the corresponding volume 2.809 cu. ft. and the PV product 323.0. The point at which the total heat content of the 115-lb. vapor equals that of the 35-lb. vapor may be called the " isocaloric " point, its temperature location differing but a little, if at all, from that of the other isocaloric point determined by equal internal energy. Some writers use the word " isodynamic " to define this latter point. Obviously, there is a point or temperature at which the $P \times V$ product for 115-lb. vapor is exactly equal to that of the 35-lb., 70° vapor, 325.1, and this is readily found from the ammonia tables to be temperature 98°. This may be called the *Iso-PV* point or temperature.

Clearly, formula (1) of page 42 does not apply to the strictly isothermal compression or expansion of vapors. It does apply, however, to compression or expansion to the *Iso–PV* point. In the example just given, for instance, it can be applied to find the work done in compressing the pound of ammonia from 35 lbs. pressure and 70° temperature up to 115 lbs. pressure and the temperature as at the *Iso–PV* point, 98°. Thus we have

Work of compression

$$= \frac{325.1 \times 2.3026 \times \log\left(\frac{115}{35}\right) \times 144}{777.5} = 71.8 \ B.t.u.$$

This is not isothermal compression, of course. It may be called *Iso–PV* compression. It is evident that formula (2) of page 43

is not usable for vapors, as it includes the non-constant factors T and R; nor is it practicable here to find the work done by multiplying the change in entropy by the absolute temperature, as the latter is non-constant.

Adiabatic Compression. — When a gas or vapor is compressed adiabatically, the initial and final volumes are not in the same ratio as that of final to initial pressure, because all of the heat of compression remains in the gas and its effect there is to cause increase in volume as well as in temperature. Whereas in isothermal or $Iso-PV$ changes we have the relation

$$PV = a \; constant \; number,$$

when dealing with adiabatic changes we have this relation:

$$PV^y = a \; constant \; number.$$

The exponent y is the ratio between the specific heat values of the gas or vapor. (See page 14. Also Table I.) The derivation of this equation will not be attempted here, but may be found in any complete work on thermodynamics. It is true for the so-called ideal gases only and not true for them at extremes of pressure or temperature. It is only approximately true for the common vapors at ordinary working pressures.

As the product of PV^y is constant for unit weight of a gas, it follows that for any specified pressures, as P_e and P_c, we may write the equation

$$P_e V_e{}^y = P_c V_c{}^y,$$

and by use of this any one of the four factors may be found when the others and the value for exponent y are known, thus:

$$P_c = \frac{P_e V_e{}^y}{V_c{}^y} = P_e \left(\frac{V_e}{V_c}\right)^y \quad (3) \qquad P_e = P_c \left(\frac{V_c}{V_e}\right)^y. \quad (4)$$

$$V_c = \sqrt[y]{\frac{P_e V_e{}^y}{P_c}} = V_e \sqrt[y]{\frac{P_e}{P_c}} \quad (5) \qquad V_e = V_c \sqrt[y]{\frac{P_c}{P_e}} \quad (6)$$

Further, these equations may be combined with the general equation stated on page 12 in such manner as to yield the following, which permit finding temperature when other conditions are known:

$$T_c = T_e\left(\frac{V_e}{V_c}\right)^{y-1}. \quad (7) \qquad T_c = T_e\left(\frac{P_c}{P_e}\right)^{\frac{y-1}{y}}. \quad (8)$$

$$T_e = T_c\left(\frac{V_c}{V_e}\right)^{y-1}. \quad (9) \qquad T_e = T_c\left(\frac{P_e}{P_c}\right)^{\frac{y-1}{y}}. \quad (10)$$

In the preceding equations,

T_e = *temperature when expanded, or before compression, and*
T_c = *temperature when compressed, or before expansion.*

The several formulas apply to expansion as well as to compression.

It will have been noted that all of the formulas or equations for adiabatic changes involve fractional roots or exponents, and these are troublesome even to one accustomed to the use of logarithms. In many of the practical problems which one meets in practice the initial and final pressures are known, and their ratio is found in equations (5), (6), (8), and (10). It is apparent that tables can be computed for a wide range of values of the term, $\sqrt[y]{\dfrac{P_c}{P_e}}$, as used in equation (5), also for the term $\left(\dfrac{P_c}{P_e}\right)^{\frac{y-1}{y}}$, as used in equation (8). Tables II and III have been prepared for these cases.

As an example showing the utility of Tables II and III, let us take oxygen gas at the same initial conditions assumed for the problem of page 43 and compress it adiabatically to same pressure; that is, starting at 70° temperature and 20 lbs. absolute pressure, it is to be compressed to 90 lbs. pressure and a temperature which we are to find. The pressure ratio is 4.5, and the value for exponent y is 1.400, as given in Table I. The temperature after adiabatic compression is given by equation (8). We find in Table III that the value of ratio 4.5 raised to the $\dfrac{y-1}{y}$ power is 1.537, and this multiplied by the initial absolute temperature, 529.6°, gives the temperature of the gas after adiabatic compression, 813.8° absolute or 354.2° ordinary temperature. Before using Table II to find the final volume per pound, we must find the initial volume, thus,

$$V = \frac{RT}{P} = \frac{48.283 \times 529.6}{20 \times 144} = 8.8787 \ \ cu. \ ft.$$

Then we multiply this initial volume by the factor found in Table II for ratio 4.5 and $y = 1.40$, thus,

$$8.8787 \times 0.3415 = 3.032 \ \ cu. \ ft.,$$

which is the final volume of the gas per pound at 354.2° temperature.

For cases in expansion the tabulated factors are used as divisors instead of multipliers. For instance, if a compressed air locomotive be working with air which, at the instant when expansion begins in its cylinders (at point of cut-off) is at 135 lbs. pressure, absolute, and 90° temperature, and the exhaust is against atmospheric pressure of 15 lbs., what is the volume per pound and the temperature of the exhaust, if the expansion be strictly adiabatic? (It could not be adiabatic in practice, on account of heating by friction and heat received from the metal of the locomotive, the outside air, etc.) The initial volume per pound is first computed and found to be 1.507 cu. ft. Then using Tables II and III, the volume of the exhaust air per pound is found to be 1.507 0.2082 = 7.24 cu. ft., while its temperature is 549.6 1.874 = 293.3° absolute, or − 166.3° ordinary temperature.

When a gas or vapor is compressed or expanded adiabatically, its entropy value remains unchanged; hence to find the temperature or heat content after compression of a gas or vapor for which a Mollier chart is at hand, it is only necessary to trace the line for entropy which passes through the initial point up to its intersection with the required final temperature at the final pressure. For example, referring to the ammonia chart, Figure 5, let ammonia at 30 lbs. pressure and 30° temperature be compressed adiabatically to 180 lbs. Locate on the chart the point for initial pressure and temperature and follow the entropy curve which passes through that point (or the path where the curve would be if drawn) up to the final pressure line, and there read the temperature of the vapor as at the instant compression ceases. It is found to be 277°. The

TABLE II

FACTORS FOR VOLUME CHANGE IN POLYTROPIC COMPRESSION OR EXPANSION

Pressure Ratio	Values for exponent n or y						
	1.20	1.25	1.28	1.30	1.33	1.40	1.66
20	.08237	.09103	.09628	.09982	.1052	.1177	.1646
19	.08597	.09484	.1002	.1039	.1092	.1220	.1697
18	.08994	.09904	.1046	.1082	.1139	.1269	.1751
17	.09432	.1036	.1092	.1131	.1188	.1321	.1814
16	.09922	.1088	.1147	.1185	.1245	.1380	.1882
15	.1047	.1147	.1206	.1246	.1306	.1445	.1957
14	.1109	.1212	.1273	.1313	.1375	.1518	.2040
13	.1180	.1284	.1348	.1390	.1454	.1601	.2132
12	.1261	.1370	.1435	.1478	.1544	.1694	.2239
11.5	.1306	.1417	.1484	.1528	.1594	.1747	.2296
11	.1355	.1468	.1536	.1581	.1648	.1803	.2358
10.5	.1409	.1524	.1592	.1639	.1707	.1864	.2425
10	.1468	.1585	.1655	.1701	.1771	.1931	.2498
9.5	.1532	.1651	.1723	.1770	.1841	.2003	.2577
9	.1603	.1725	.1797	.1845	.1917	.2082	.2662
8.8	.1633	.1756	.1828	.1877	.1949	.2114	.2698
8.6	.1665	.1788	.1862	.1911	.1984	.2151	.2736
8.4	.1697	.1823	.1896	.1945	.2018	.2186	.2774
8.2	.1732	.1858	.1932	.1981	.2055	.2224	.2815
8.0	.1768	.1895	.1970	.2019	.2094	.2264	.2857
7.8	.1806	.1933	.2009	.2059	.2135	.2305	.2901
7.6	.1845	.1974	.2050	.2101	.2177	.2348	.2946
7.4	.1887	.2017	.2094	.2145	.2221	.2393	.2995
7.2	.1930	.2061	.2138	.2190	.2267	.2441	.3045
7.0	.1975	.2107	.2186	.2238	.2315	.2489	.3095
6.8	.2024	.2157	.2237	.2289	.2367	.2543	.3151
6.6	.2075	.2210	.2290	.2342	.2419	.2597	.3207
6.4	.2129	.2266	.2345	.2398	.2477	.2656	.3269
6.2	.2186	.2324	.2404	.2458	.2537	.2717	.3332
6.0	.2247	.2384	.2466	.2520	.2599	.2781	.3398
5.8	.2312	.2452	.2533	.2587	.2668	.2849	.3469
5.6	.2380	.2521	.2603	.2658	.2739	.2921	.3542
5.4	.2453	.2595	.2678	.2733	.2815	.2998	.3621
5.2	.2531	.2674	.2758	.2813	.2896	.3080	.3704
5.0	.2615	.2758	.2843	.2898	.2982	.3166	.3792
4.8	.2707	.2852	.2937	.2992	.3075	.3262	.3888
4.6	.2803	.2949	.3035	.3091	.3175	.3361	.3987
4.4	.2909	.3057	.3143	.3199	.3283	.3470	.4096
4.2	.3025	.3174	.3260	.3316	.3400	.3588	.4213
4.0	.3149	.3298	.3355	.3442	.3526	.3715	.4338
3.8	.3287	.3438	.3524	.3580	.3665	.3853	.4474
3.6	.3440	.3589	.3676	.3733	.3818	.4005·	.4622
3.4	.3607	.3757	.3843	.3900	.3985	.4173	.4784
3.2	.3794	.3944	.4030	.4087	.4172	.4357	.4963
3.0	.4003	.4153	.4239	.4295	.4378	.4562	.5159
2.8	.4239	.4387	.4474	.4528	.4612	.4792	.5376
2.6	.4511	.4656	.4741	.4794	.4875	.5054	.5623
2.4	.4821	.4964	.5047	.5099	.5177	.5351	.5900
2.2	.5184	.5322	.5401	.5453	.5529	.5696	.6219
2.0	.5613	.5744	.5819	.5866	.5939	.6095	.6588

initial total heat content is read as 629.0 B.t.u., the final total heat as 752.0 B.t.u. per pound. If we use Figure 6 the final temperature should agree with that read from Figure 5, 277°; the initial internal energy at the beginning of compression is 573.5 B.t.u., and the final internal energy of the compressed vapor is 670.0 B.t.u. per pound.

Work of Adiabatic Compression. — The work done on the gas or vapor during the compressive part of the piston stroke, or the work done against the piston by the expanding gas or vapor during expansion, is as stated by the following formula:

$$Work, \ in \ foot\text{-}pounds = \frac{P_c V_c - P_e V_e}{y - 1}. \qquad 11)$$

The mathematical derivation of this formula is to be found in complete treatises on thermodynamics. The pressures, P_c when compressed and P_e when expanded or before compression, are in pounds per square foot, absolute. Volumes V_c and V_e are in cubic feet per pound. The work as found by the formula is in foot-pounds and for one pound weight of the gas or vapor. If the pressures as used are per square inch, absolute, the result found therewith must be multiplied by 144. If the heat equivalent of the work be desired, the result in foot-pounds must be divided by 777.5.

Formula (11) gives the work of compression only, corresponding to that measured by the area under curve a-b in (B) of Figure 9. To obtain the net total work for the complete piston stroke the expulsive work, which is $P_c V_c$, must be added, and the contra-work done against the back face of the piston by the incoming low-pressure gas must be subtracted, it being $P_e V_e$. However, it can be proved by a simple algebraic process that the net total work as would thus be found is equal to the work of compression multiplied by the value of exponent y. That is, the net total work of the piston stroke, per pound of gas compressed, is expressed by this formula:

$$Net \ total \ work = y \frac{P_c V_c - P_e V_e}{y - 1}. \qquad (12)$$

TABLE III

FACTORS FOR TEMPERATURE CHANGE IN POLYTROPIC COMPRESSION OR EXPANSION

Pressure Ratio	Values for exponent n or y						
	1.20	1.25	1.28	1.30	1.33	1.40	1.66
20	1.647	1.820	1.925	1.995	2.103	2.353	3.291
19	1.633	1.802	1.904	1.972	2.076	2.319	3.224
18	1.619	1.782	1.882	1.948	2.048	2.283	3.156
17	1.603	1.762	1.858	1.923	2.019	2.247	3.085
16	1.587	1.741	1.833	1.895	1.989	2.208	3.011
15	1.570	1.719	1.808	1.868	1.958	2.167	2.936
14	1.552	1.695	1.781	1.839	1.925	2.125	2.856
13	1.533	1.670	1.752	1.807	1.906	2.081	2.773
12	1.513	1.644	1.722	1.774	1.852	2.033	2.686
11.5	1.503	1.629	1.706	1.757	1.833	2.009	2.641
11	1.491	1.615	1.690	1.740	1.813	1.985	2.595
10.5	1.480	1.601	1.673	1.721	1.793	1.957	2.548
10	1.468	1.583	1.655	1.701	1.770	1.931	2.499
9.5	1.455	1.569	1.636	1.681	1.748	1.903	2.447
9	1.442	1.552	1.616	1.661	1.725	1.874	2.395
8.8	1.436	1.545	1.609	1.652	1.716	1.862	2.374
8.6	1.431	1.538	1.601	1.643	1.705	1.849	2.353
8.4	1.426	1.531	1.592	1.634	1.696	1.837	2.331
8.2	1.420	1.524	1.584	1.625	1.686	1.824	2.309
8.0	1.414	1.516	1.576	1.615	1.675	1.811	2.287
7.8	1.408	1.508	1.567	1.606	1.665	1.799	2.263
7.6	1.403	1.501	1.559	1.597	1.654	1.785	2.240
7.4	1.396	1.492	1.549	1.587	1.644	1.771	2.216
7.2	1.390	1.485	1.540	1.577	1.632	1.758	2.193
7.0	1.383	1.476	1.531	1.567	1.621	1.744	2.168
6.8	1.377	1.468	1.521	1.557	1.609	1.730	2.143
6.6	1.370	1.458	1.511	1.546	1.597	1.716	2.118
6.4	1.362	1.450	1.501	1.535	1.585	1.699	2.092
6.2	1.355	1.441	1.491	1.524	1.572	1.684	2.066
6.0	1.348	1.431	1.480	1.513	1.560	1.668	2.039
5.8	1.340	1.421	1.469	1.501	1.547	1.652	2.012
5.6	1.332	1.411	1.457	1.488	1.533	1.636	1.984
5.4	1.324	1.401	1.445	1.476	1.520	1.620	1.955
5.2	1.316	1.391	1.434	1.463	1.505	1.602	1.926
5.0	1.308	1.380	1.422	1.450	1.490	1.584	1.896
4.8	1.299	1.369	1.409	1.436	1.475	1.565	1.866
4.6	1.290	1.357	1.396	1.422	1.460	1.546	1.835
4.4	1.280	1.345	1.383	1.408	1.444	1.527	1.803
4.2	1.270	1.332	1.369	1.393	1.428	1.507	1.770
4.0	1.260	1.319	1.355	1.377	1.411	1.486	1.736
3.8	1.249	1.306	1.340	1.361	1.393	1.464	1.700
3.6	1.238	1.292	1.324	1.344	1.374	1.442	1.664
3.4	1.227	1.277	1.307	1.326	1.355	1.419	1.627
3.2	1.214	1.262	1.290	1.307	1.335	1.395	1.588
3.0	1.201	1.246	1.272	1.288	1.313	1.369	1.547
2.8	1.187	1.229	1.252	1.268	1.291	1.342	1.506
2.6	1.173	1.211	1.232	1.247	1.268	1.314	1.462
2.4	1.157	1.191	1.212	1.224	1.243	1.284	1.417
2.2	1.140	1.170	1.188	1.199	1.216	1.252	1.368
2.0	1.122	1.148	1.163	1.173	1.188	1.219	1.317

Moreover, as at any particular temperature and for an ideal gas we have the relation, $PV = RT$, it follows that the corresponding values for R and T may be substituted in formulas (11) and (12), which then become:

$$Work\ of\ compression = \frac{RT_c - RT_e}{y - 1} = R\,\frac{T_c - T_e}{y - 1}. \quad (13)$$

$$Net\ total\ work = yR\,\frac{T_c - T_e}{y - 1}. \quad (14)$$

It is not practicable to use formulas (13) and (14) for vapors, as the value of R is not constant for a vapor.

As an example in finding the work done in adiabatic compression, we shall take the same initial data as used for isothermal compression of oxygen, page 43, and compare the results obtained. The temperature after adiabatic compression has been found to be $354.2°$ (page 47) and the volume per pound 3.032 cubic feet. Then, applying formula (11), the work of the compressive part of the stroke is

Work of compression
$$= 144 \times \frac{90 \times 3.032 - 20 \times 8.879}{0.40} = 34,308\ \textit{foot-pounds},$$

the heat equivalent of which is, $34,308\ 777.5 = 44.13$ *B.t.u.*, and the net total work of the piston is, per pound of gas,

$$34,308 \times 1.40 = 48,031\ \textit{foot-pounds},\ \text{or}$$
$$48,031/777.5 = 61.78\ \textit{B.t.u.}$$

It may be seen that this is considerably more than the 49.47 B.t.u. work found for isothermal compression.

It is easy to verify the fact that multiplying the compression work by the value of exponent y gives the same result as obtained by adding together the work of expulsion and compression and subtracting from their sum the contra-work done against the piston by the incoming low-pressure gas. The work of expulsion, P_cV_c, is in this example, $90 \times 144 \times 3.032 = 39,295$ *foot-pounds*, while the contra-work is P_eV_e, or

$20 \times 144 \times 8.879 = 25,572$ *foot-pounds.* Then we have $34,308 + 39,295 - 25,572 = 48,031$ *foot-pounds,* as before.

If we choose to apply formula (14), for comparison, we have

Net total work

$$= 48.283 \times 1.40 \times \frac{354.2 - 70}{0.40} = 48,027 \text{ } foot\text{-}pounds,$$

which is the equivalent of 61.77 B.t.u. of heat energy.

When a gas or vapor is compressed adiabatically, all of the heat of compression remains in the gas, the internal energy content of the gas being thus increased by the heat equivalent of the work of compression. It follows that if for any case in adiabatic compression we can determine the internal energies of the gas as before and after the compression, their difference represents the work of compression expressed in B.t.u. of equivalent heat. Given a special Mollier chart on the order of Figures 6, 8, and 18, or a table showing internal energy content, it is an easy matter to read therefrom the initial and final energies, subtract, and thus obtain the B.t.u. equivalent of the work of compression (the work of only the compressive part of the piston stroke). The Mollier chart may be dispensed with when dealing with an ideal gas, as in that case the change in internal energy is readily found by multiplying the change in temperature by the value for C_v (specific heat at constant volume) for the gas considered. The latter method gives

Work of compression, in B.t.u. $= (354.2 - 70) \times 0.1552 = 44.\mathbf{11},$

agreeing with the result first obtained.

It has been explained that after the work of compression is found, it is only necessary to multiply it by the value of y to obtain the net total work. [Formulas (12) and (14).] But it had been shown previously (pages 22 and 23) that the gain in internal energy multiplied by the value of y is the gain in total heat. Therefore, as the work done in adiabatic compression is the exact equivalent of the internal energy gained by the gas during compression, the total heat gained during compression necessarily must be the heat equivalent of

the net total work of the piston, it being equal to work of
compression multiplied by y. The change in total heat dur-
ing compression is obtainable from any standard Mollier chart
or the tables of properties, for a vapor, or in the case of a gas
simply by multiplying the difference between the initial and
the final temperatures by the value of C_p which pertains to
the gas, C_p being the specific heat at constant pressure. This
heat-difference method for finding either the work of compres-
sion or the net total work, for the case of adiabatic compression,
gives for vapors results more accurate than obtainable through
use of formulas (11) and (12) and identical results for the
perfect gases. For the example, the net total work by the
heat-difference method is

$$0.2173 \times (354.2 - 70) = 61.76 \ B.t.u.$$

Perhaps emphasis should be laid on the fact that the state-
ments as to equivalence of work done to change in heat content
apply only to the case of adiabatic compression. At any final
temperature other than the adiabatic, the difference between in-
itial and final total heat contents is not the heat equivalent of
the net total work, nor is the difference between initial and final
internal energies the heat equivalent of the work of the com-
pressive part of the stroke. When the temperature of the gas at
the instant compression ceases is more or less than the adiabatic,
either heat has been added to the gas from some source other
than the work of compression or some of the heat of compres-
sion has been lost from the gas to an outside substance, the final
heat content of the gas giving no clue as to the magnitude of
that gain or loss.

Work of Polytropic Compression. — The various heat gains
and losses in ordinary compression of gases, air or vapors, or in
expansion as in an air engine, the steam engine, etc., conspire
to make the compression or expansion polytropic in character
(page 38). The relations among volumes, pressures, and tem-
peratures incident to polytropic changes can be expressed as
equations similar in form to equations (3) to (14), inclusive, in
all cases substituting for y another exponent usually designated

by the letter n. The value of n varies with conditions, depending on what portion of the heat of compression is permitted to remain in the gas until compression ceases, on the quantity of heat received from external sources, and, in the case of a vapor, on the initial and (or) final quality. If no portion of the heat of compression remains in the gas, and if no heat be received by it from other sources, then the value for n is 1.00, this being the case of isothermal or *Iso-PV* compression. In this case $n - 1 = 0$, the intromission of the factor 0 making the several equations named inapplicable. If more heat than that of compression be abstracted from the gas during the compressive part of the stroke, with no gain from external source to compensate for the loss, then the value for n is less than 1.00; if none of the heat of compression be lost and no other heat be added to or taken from the gas during compression, we have the adiabatic case and n is equal to y; if all of the heat of compression remains in the gas and in addition it receives heat from some outside source, such as friction of piston rings against cylinder metal, then the value of n becomes greater than that of y; while between the adiabatic and the isothermal condition, the common operating case, the value of n is somewhere between that of y and 1.00.

Given the initial and the final absolute pressures and the corresponding volumes per pound, in any case of compression or expansion the value of n is findable by use of this formula:

$$n = \frac{\log P_c - \log P_e}{\log V_e - \log V_c}. \tag{15}$$

The symbols have same meanings as in formulas previously written, viz.:

$P_e = $ *pressure before compression, or after expansion,*

$P_c = $ *pressure after compression, or before expansion,*

$V_e = $ *volume per lb. before compression or after expansion, and*

$V_c = $ *volume per lb. after compression or before expansion.*

Pressures may be pounds per square foot or per square inch.

Now let us take a case in polytropic compression, using one pound of oxygen gas as in former examples, the initial tempera-

ture being 70°, initial pressure 20 lbs., and final pressure 90 lbs.
absolute. Let the operating conditions be such that the tem-
perature of the gas as at the instant when compression ceases is
240°. Before we can use formula (15) we must find the volume
per pound of the compressed gas, which is:

$$V = \frac{RT}{P} = \frac{48.283 \times 699.6}{90 \times 144} = 2.604 \ cubic \ feet.$$

Then, with the aid of a table of common logarithms, the several
values are substituted in formula (15), with this result:

$$n = \frac{1.9542 - 1.3010}{.9483 - .4157} = \frac{.6532}{.5326} = 1.226.$$

Having now the proper value for n, we can use it in formula (11)
to find the work of compression, thus:

Work of compression

$$= \frac{144 \ (90 \times 2.604 - 20 \times 8.879)}{777.5 \times 0.226} = 46.53 \ B.t.u.,$$

while the net total work by formula (12) is

$$46.53 \times 1.226 = 57.04 \ B.t.u. \ per \ pound \ of \ oxygen \ compressed.$$

The General Graph for Gas Compression. — On page 43 we
found the work done in the isothermal compression of a pound of
oxygen gas from an initial pressure of 20 lbs. to a final pressure
of 90 lbs. per square inch, the temperature being constant at 70°
throughout the process. On page 52 we found the work of com-
pression, also the net total work, for a pound of oxygen com-
pressed adiabatically to temperature 354.2°, and on page 56 the
work of polytropic compression to a specified temperature, 240°.
The method used for the last-named case can be used for any in
polytropic compression, and next we shall apply it through the
entire range of final temperatures, plot the results so obtained,
and draw therefrom such conclusions as we may.

We compute the volumes of the compressed oxygen gas (the
initial conditions and pressure ratio being those mentioned in
the preceding paragraph), the several values for n and the work

of compression in B.t.u. of heat equivalent for a series of final temperatures, up to a very high point, 2800°, and in the other direction for temperatures down to the limit, absolute zero temperature, and plot the results, thereby determining the curve $C-V-P-D$ of Figure 10. In this the base line $A-B$ represents the internal energy content of the pound of gas at the initial temperature and pressure. Final gas temperatures are marked by
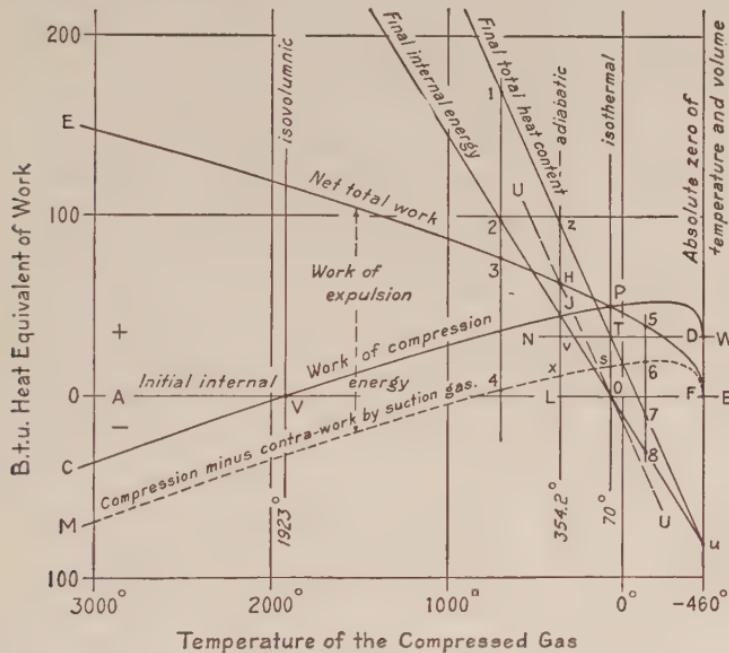
FIG. 10. THE GENERAL GRAPH FOR GAS COMPRESSION

the horizontal scale and the B.t.u. of heat equivalent of work done by the vertical scale. It is to be understood, of course, that oxygen gas could not actually be brought up to the higher temperatures used in the graph, because fusion of the compressor metal, or some other change, would intervene. The graph serves, nevertheless, to illustrate the laws governing compression. As the next step, we find the net total work as for every one of the series of final temperatures, thereby determining a series of points through which is drawn the net total work curve, $E-H-P-F$. This intersects the compression curve on the iso-

thermal temperature vertical, 70°, as at that temperature the
total work is neither more nor less than the work of compression.
At the adiabatic temperature, 354.2°, the net total work, as
measured from L to H, is equal to the work of compression
multiplied by the value for n, which at that particular point is
the same as that of y; that is, distance $L–J$ multiplied by y
equals $L–H$. As the final temperature increases, the work of
compression becomes less and less, because of the reception from
outside sources of heat which does part of the work of raising
the pressure, the final volumes increasing the while, until even-
tually a point is reached, at about 1923° temperature, where no
mechanical effort is required to raise the pressure of the gas from
20 lbs. to 90 lbs. At that final temperature all work would be
done by heat, there being no change in volume of the gas. Just
as on page 10 it was seen how the pressure doubled when the
temperature rose through 492° in accordance with Charles's law,
the volume remaining constant, here the final pressure is 4.5
times the initial in obedience to that same law, the final ab-
solute temperature being 4.5 times the initial, and with no
change in volume. We are assuming here that Charles's law
holds good at extremes of temperature, which is not strictly
true.

At the isothermal point the value of n is 1.00. At the adia-
batic it is 1.40. At temperatures progressively higher than the
adiabatic the value of n becomes greater and greater until, at
point V, which may be called an " isovolumnic " point (it
being one point at which the final and initial volumes are equal),
the value for n is an infinitely large number. Upon the con-
tinued application of heat and the raising of the final tempera-
ture higher than that at the isovolumnic point, the volume
must become greater than at the beginning, as otherwise there
would be a rise in pressure above the 90 lbs. limit set for the
problem. The volume thus becoming larger instead of smaller
as in normal compression, the work done is negative instead
of positive, and is measured below the base $A–B$. Thus, at
2600° it is, by scale, about -22 B.t.u. in heat equivalent.
Moreover, values of n for final temperatures above that at

the isovolumnic point are negative and vary from infinity at that point down to 0.0 at infinitely high temperature. At the latter limit the work of compression, and likewise the net total work, is infinitely great. For final temperatures lower than the isothermal the value of n varies from 1.00 at that point to 0.0 at the absolute zero of temperature, taking successively every value between these limits.

Throughout the entire range of final compression temperatures, both practicable and speculative, from the absolute zero as one limit to infinity as the other, the contra-work done against the back face of the piston by the incoming low-pressure gas is a constant quantity. It is P_eV_e, the numerical value of this product in the case in hand being 25,571 foot-pounds, the mechanical equivalent of 32.89 B.t.u. of heat. Measuring this constant heat quantity downward from curve $C–V–P–D$ on the vertical lines marking temperatures, we determine the locations of points through which the curve $M–x–s–F$ is drawn. Now at any temperature (of the compressed gas as at the instant when compression ceases) the distance between curve $M–x–s–F$ and curve $E–H–P–F$ measures the work done in expelling the compressed gas from the cylinder (measuring on the vertical temperature line in every instance). Thus at 70° it is equal in amount to the contra-work, 32.9 B.t.u., marked as $P–s$; at the isovolumnic point, it is exactly 4.5 times the contra-work, or 148 B.t.u., and is 0.0 at absolute zero temperature. At that point the volume of the compressed gas is 0.0, and no work is required or possible for its expulsion. This, it may be noted here, is true whatever the final pressure to which the gas is raised. If the final pressure were 200 lbs., or 3000 lbs., or anything whatsoever, the work of expulsion at absolute zero temperature still would be 0.0.

The point marked as D is at distance 32.9 B.t.u. above point F, meaning that the work of compression to absolute zero temperature is exactly equal to the contra-work done by the low-pressure gas entering the cylinder. Referring back to Figure 9, in this case the curve $a–b$ covers the full length of the stroke and point b coincides with point c.

We next lay on the diagram curves marking the internal energy and the total heat contents of the pound of oxygen gas as at the several final temperatures. As we assume that there is no variation in the specific heats for oxygen gas, even at extremes of temperature, these two curves plot as straight lines radiating from a point on the absolute zero temperature vertical. The curve for internal energy passes through points O and J, which mark the internal energy content of the gas as before and after compression adiabatically. Point O marks the intersection of the initial internal energy line, $A–B$, the curve for final internal energy, and the 70° temperature vertical. Point J marks the intersection of the curve of internal energy, the compression curve, and the adiabatic temperature vertical. Point H marks the intersection of the net total work curve, the adiabatic temperature vertical, and a straight line drawn through point O parallel to the final total heat line. If now we find the initial total heat content of the pound of oxygen when at 70°, it being 115.1 B.t.u. as reckoned above absolute zero heat content, and lay on the diagram a horizontal line, $N–W$, marking this heat content, it is found that it passes through point D, exactly 32.9 B.t.u. above the 82.2 initial internal energy content marked by line $A–B$. The 32.9 B.t.u. represents external work done, equal in amount to the contra-work done by incoming low-pressure gas. The line $U–U$, drawn parallel to final total heat line, must pass through point H, because $z–v = H–L$, and $z–H = T–O$. $H–L$ is the gain in total heat content during adiabatic compression. The following equations are true:

$$J-x = D-F = T-O = H-z = P-s = v-L = \frac{144 \times P_e \times V_c}{777.5}.$$

Furthermore, at any final temperature, the difference between total heat and internal energy, which is a measure of external work, is equivalent to the work of expulsion. At the absolute zero this work is 0.0; at $-140°$ it is measured at 5–6, which is equal to 7–8; at the isothermal point, $P–s = T–O$, as stated before, and the external work is the same as that for the low-

pressure gas; at the adiabatic, $z-J = H-x$, at 700°, we have $1-2 = 3-4$, etc.

With the pressures used in our problem, the work of compression apparently is at maximum between $-200°$ and $-300°$ final temperature, and falls off sharply in the last half-hundred degrees. At those very low temperatures the value of n is approaching zero. Values of n for our series of computations are plotted in Figure 11. The curve to the right, for positive values of n, would meet the isovolumnic axis at an infinite distance
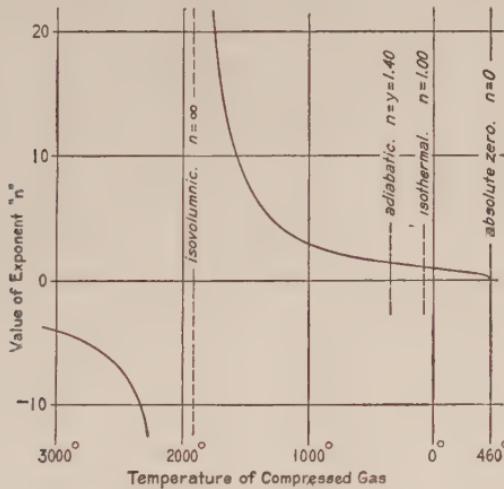
FIG. 11.   PLOTTED VALUES OF EXPONENT "n" FOR TYPICAL
CASE OF COMPRESSION

above axis 0–0, and intersects axis 0–0 at absolute zero temperature. The curve for the negative values of n, to the left, would meet the isovolumnic axis at an infinite distance below axis 0–0 and meet axis 0–0 when prolonged an infinite distance to the left, to temperature infinity.

Returning to Figure 10, it is apparent that the work of compression or the net total work for any final temperature can be found by scaling the distance to the curve from base line $A-B$. Moreover, if the net total work be divided by the work of compression for any temperature, the quotient is the value for n as at that temperature.

When the one pound of oxygen gas is compressed adiabatically, its final volume, 3.032 cu. ft., is 34.15 per cent of its initial volume; when compressed isothermally, its final volume is 22.22 per cent of the initial; compressed to zero volume at absolute zero temperature, its final volume is 0.0 per cent of the initial; at the isovolumnic point, the final volume is equal to the initial, or is 100 per cent; and when compressed by heat alone to still higher temperatures, the volume becomes larger than at beginning, becoming infinity at infinite temperature. Between the point where volume is 100 per cent of the initial and the point where the volume becomes infinity, the piston is forced backwards by expanding gas instead of advancing against the gas as in normal compression. Figure 12 may be of service in this connection. Starting with the cylinder full of gas, when the piston has advanced to point ba, the curve representing the gradually rising pressure, from a to ba, is the adiabatic curve, for the case $n = y = 1.40$, and the volume of the compressed gas is 34.15 per cent of that when compression started. At the isothermal point where $n = 1.00$, the piston face has advanced to bi, and the volume of the compressed gas is only 22.22 per cent of that at starting. The lower the final gas temperature, the smaller the volume, and the farther the piston travels before bringing the gas to a specified pressure.

The values of exponent n, starting with 1.00 at the isothermal point, vary to 1.40 at the adiabatic point, and from that to infinity at the isovolumnic point; then from infinity to the value -0 at infinitely high temperature, for which imaginary condition the compression curve is a straight horizontal line coinciding with line $s-a$. In sub-isothermal compression the value of n decreases from 1.00 at isothermal to 0.0 at the absolute zero temperature, the sharpness of the curve gradually becoming more marked until at the absolute zero the curve is a right angle coincident with lines $a-g-c$. In all cases the work of compression is measured by the area under the compression curve. Thus at the adiabatic it is area $a-ba-ea-f$; where $n = 2$, it is $a-v-x-f$; where $n = 0.80$, it is $a-w-i-f$; where $n = 10$, it is $a-k-h-f$; where $n = $ infinity, the isovolumnic point, the curve is a verti-

cal line, and there is no area under it, meaning that the work of compression is 0.0. Areas under compression curves to the left of this point measure negative work done when the piston is forced back beyond its normal starting point by heating of the gas, as in the case when $n = -5.0$, for instance, where the face of the piston must recede to point marked t. At absolute zero temperature the area under the compression curve is
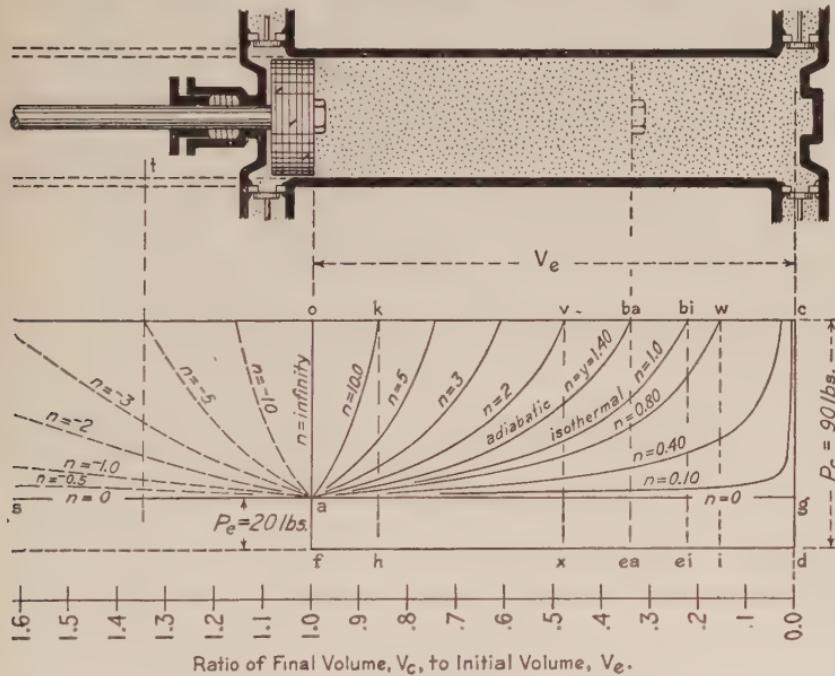
FIG. 12.   PRESSURE-VOLUME RELATIONS IN COMPRESSION

a–g–c–d–f, equal to area a–g–d–f, which measures the contra-work done against the piston by incoming low-pressure gas. In any and all cases the work as measured by the area under the compression curve must agree, when converted into B.t.u., with the B.t.u. as scaled between line A–B and the compression curve of Figure 10.

The item of contra-work is constant, regardless of the location of the compression curve. By the area method it is area a–g–d–f, Figure 12, or by the heat-difference method it is the constant

distance, in B.t.u., between curves C–V–P–D and M–x–s–F, Figure 10. It is the equivalent of the initial volume multiplied by the initial pressure, $V_e P_e$, as for a specified initial temperature of gas, and is independent of final pressure and temperature.

The work of expulsion is measured by the area of the rectangle between the upper end of the compression curve and line c–d. For the adiabatic compression, it is ba–c–d–ca; for the isothermal, bi–c–d–ei; for $n = 0.80$, it is w–c–d–i; for $n = 0$, it is the straight line c–d, without area; where $n = 10$, it is k–c–d–h;
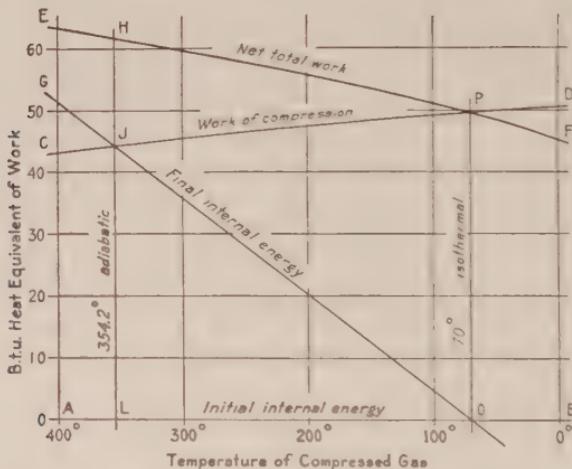
FIG. 13. NORMAL WORK CURVES FOR COMPRESSION

where $n = $ infinity, it is o–c–d–f, and to the left of this, where final volumes exceed the initial, the work of expulsion is greater and greater until it becomes infinitely great at infinite temperature.

When a gas or vapor is compressed to a moderate pressure, it is usually found that the work of compression is less at the adiabatic discharge temperature than at the isothermal. This is true of the conditions depicted by Figure 10, the work curve between points J and P being an ascending curve. The work of compression to a high pressure, on the other hand, may be greater at the adiabatic than at the isothermal point. The net total work is always greater at the adiabatic than at the iso-

thermal, the curve from H to P being in any case a descending curve.

It is seen in Figure 10 that the work curves between adiabatic and isothermal points are nearly straight lines. Figure 13 shows to a larger scale this part of the curves. They have been drawn carefully, in order to show how little is the curvature in this particular case. If one should make the necessary series of computations for any practicable case of compression for any gas or vapor and plot the curves and the corresponding chords which pass through points J and P, and H and P, it would be found that at the temperature where variation was greatest the difference between the work as measured to the curve and that if measured to the chord would hardly exceed one per cent. In the compression of air, ammonia, and similar vapors to ordinary working pressures the difference is small, usually less than one per cent. The fact that the two work curves may be replaced thus by straight lines is important, since it makes practicable the use of a quickly made graphical analysis for many problems in compression and expansion.

Heat-Work Diagram, Compression. — We take a problem in air compression to illustrate this graphical method of analysis. Let the low-pressure air be at atmospheric pressure and 50° temperature after passing the inlet valves of the compressor, and let the ratio of compression be 6, that is, from 14.7 lbs. up to 88.2 lbs. absolute. Following methods which have been explained in detail in the solution of previous problems, we first find the initial volume of the air, per pound, and then the heat equivalent of the work for isothermal discharge temperature, it being 62.6 B.t.u. Next we find the temperature as at end of adiabatic compression, 390.4°, and the work of compression and the net total work at that temperature. These are, respectively, 58.2 and 81.6 B.t.u. We now have all data necessary for drawing a simple heat-work diagram, Figure 14. First lay down the base line for initial internal energy content, A–B; then fix to any suitable scale the locations of the isothermal and adiabatic temperatures. On a vertical line through the former locate point P at 62.6 B.t.u. above A–B, using any

convenient scale for B.t.u., and on the adiabatic temperature vertical locate points J and H at 58.2 and 81.6 B.t.u. above $A–B$. Draw the straight line for final internal energy content through points J and O, and the straight lines $C–J–P–D$ and $E–II–P–F$, which are the two work "curves." These six straight lines constitute the heat-work diagram in its simplest form.

The work of compression and the net total work for any practicable compression temperature can be scaled from the
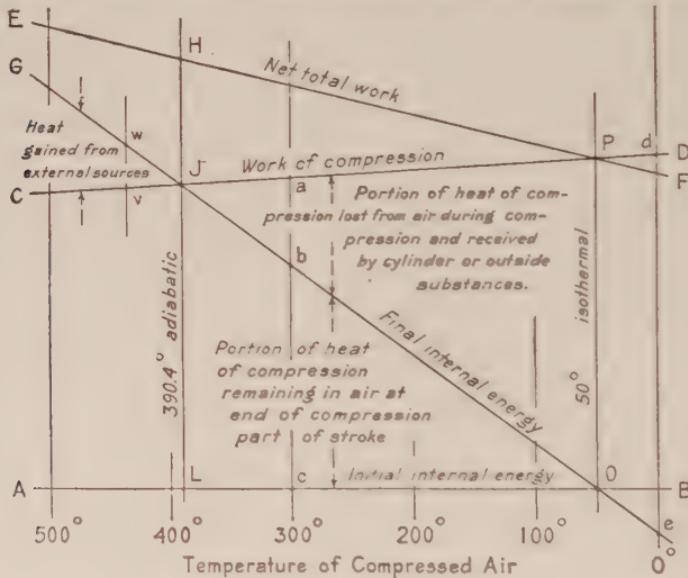
FIG. 14.　HEAT-WORK DIAGRAM FOR COMPRESSION OF GAS

heat-work diagram. For instance, if the final temperature be 300°, the work of compression, measured on the vertical line marking 300°, scales as 59.3 B.t.u., and the net total work scales as 76.4 B.t.u. Computations made for this discharge temperature yield 60 and 77 B.t.u., the error in scale reading thus being only about one per cent, inclusive of the error involved in use of straight lines in place of the work curves. This error is negligible for all practical purposes.

A study of the heat-work diagram, Figure 14, indicates that any of the heat of compression remaining in the air at the in-

stant when compression ceases can be measured by scaling the difference between initial and final internal energies on the vertical line that marks the discharge temperature. For instance, at discharge temperature 300° (strictly, the temperature of the air as at the instant when compression ceases; it may change between that instant and its passage through discharge valve), the heat of compression that remains is measured as b–c, being about 42.5 B.t.u. The heat of compression to 300° is equivalent to the work, 59.3 B.t.u.; and the difference between this and the portion that remains in the compressed air is the heat lost to jacket water, by radiation to the air of the engine room, to cold metal, to the cool air entering the cylinder, etc., such lost heat being measured as a–b, 16.8 B.t.u. At the isothermal temperature none of the heat of compression remains; all is yielded to jacket water, etc. At final temperatures less than the isothermal, the compressed air yields to jacket water, etc., not only all of the heat of compression but a portion of its initial internal energy content as well. Thus at final temperature 0° the heat carried away is measured at e–d, as 71.7 B.t.u. Of this the part above base line A–B is heat of compression, it being 63.3 B.t.u., while the part below A–B, 8.4 B.t.u., is lost from the initial internal energy content. At the adiabatic temperature all heat of compression remains in the compressed air; none is yielded to any other substance. At temperatures above the adiabatic, all heat of compression remains in the air; and in addition heat is received from some source other than compression, this additional heat being measurable between the line for final internal energy and the compression curve, vertically in every case, as for instance as at v–w for final temperature 435°, where it scales 8.8 B.t.u.

Figure 15 is another simple heat-work diagram, for ammonia vapor, the assumed initial pressure being 40 lbs., absolute, initial temperature 70°, final pressure 200 lbs. Again referring to Figures 10 and 1 , it will be observed that for those cases or for any with an ideal gas, the line marking the final internal energy is straight and may be drawn as soon as the location

of point J is determined, point O being fixed by the conditions set for the problem. For vapors which have non-constant specific heat values, the final internal energy line is not quite straight. It is easily plotted, however, with the aid of a complete table or Mollier chart showing internal energy, such as Figure 6 for ammonia or 8 for steam. Thus plotting the curve, G–O–f, it is seen that at f, which marks the condensation point, its direction changes abruptly to the vertical. The point
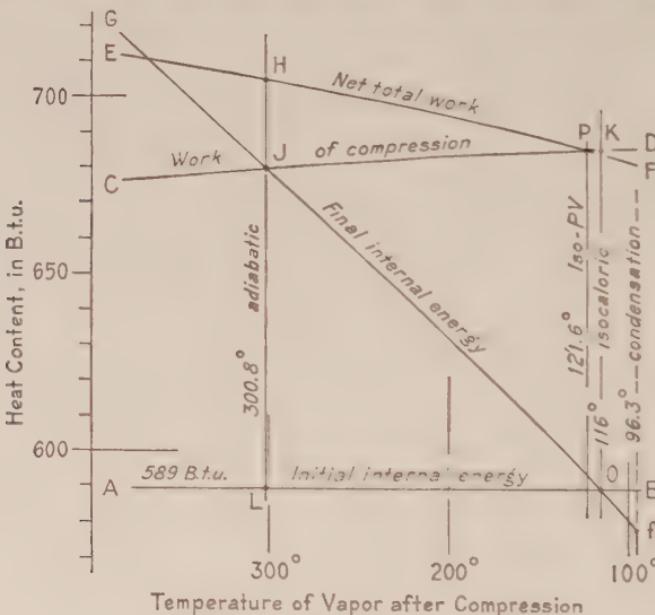
FIG. 15. HEAT-WORK DIAGRAM FOR COMPRESSION OF VAPOR

P is located on the calculated Iso-PV temperature vertical, as defined on page 45. It is the temperature at which in this instance the formula for so-called isothermal compression holds good. True isothermal compression to final temperature 70° is not practicable in this case, that temperature being below that at which condensation of the compressed vapor would begin. The two work curves represent the results obtained by a series of computations using formulas (15), (11), and (12). If compared with a straight-edge it will be seen that their curvature is slight. Moreover, if a point K be located at the

same height as point P but on a vertical line erected through point O, that is, on the isocaloric temperature vertical (page 45), it appears that a straight line drawn through points H and K more nearly coincides with the actual net total work curve than does a straight line through H and P. It follows that in certain cases where it is not convenient or possible to locate point P, we may locate point K and proceed with the diagram. In general, the use of point K is preferable to use of point P.
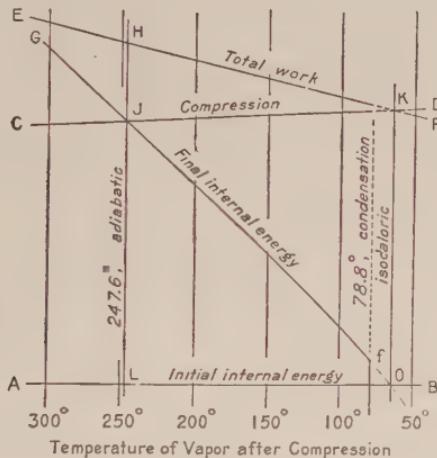
FIG. 16.   USE OF THE ISOCALORIC POINT
INSTEAD OF THE ISOTHERMAL

Figure 16 is presented to clarify the statements regarding use of point K. It is a heat-work diagram for another case in ammonia compression, from 25 lbs. to 150 lbs. pressure, the initial temperature being 10°. Plotting the final internal energy curve, to the condensation point, f, it is found that the curve does not intersect the base line, A–B. In the last example it was impossible to have isothermal compression, on account of condensation of the vapor; in the example now given, it is impossible to have isothermal, isocaloric, or Iso-PV compression without condensation. However, the work of compression with which we are concerned varies with the rate of volume reduction, and this rate is fairly constant down to the condensation point. We can, without serious error, disregard the

accident of condensation and prolong the heat curve until it intersects the initial energy line at O, and on the vertical through this point locate point K at the height determined by formula (1).

Figure 17 embodies a series of heat-work diagrams for ammonia vapor, compressed from 10° initial temperature and 20 lbs. absolute pressure up to the various pressures indicated,

FIG. 17. HEAT-WORK DIAGRAMS FOR AMMONIA VAPOR
COMPRESSED TO SEVERAL FINAL PRESSURES

100, 125, 150, 170, 200, and 250 lbs., absolute. If a similar diagram were to be made for a case with air or an ideal gas, there would be but a single line marking final internal energy, which would not vary with the pressure. For a vapor, however, there must be a separate curve for each final pressure, as heat content varies with pressure. These curves lie very close to one another, their separation being exaggerated in Figure 17. None of them crosses the base line; and in order to fix the several isocaloric points, we prolong each to cut the base line, as explained in preceding paragraph. Any error thus intro-

duced does not materially affect the work of compression as scaled for ordinary discharge temperature. It may be noted that the several J-points lie on a nearly straight line, as do also the H-points, while the K-points lie on a smooth curve, and the condensation points on another smooth curve which is in fact the saturation curve of chart Figure 6.

Figure 18 is a chart useful in the solution of problems in air compression and expansion, as explained briefly on page 36, it being possible to read therefrom the initial and final heat contents (both internal energy and total heat) and the temperature of air as after adiabatic compression or expansion. As an example in its use, suppose that in an air compressor the temperature of the air in the cylinder as compression stroke begins is 70° and that the compression ratio is 6; that is, the final absolute pressure is 6 times the initial absolute pressure. Following a vertical line through temperature 70° up to the heat scale along the top, we read there the initial heat contents (reckoned above those at − 100°) as 29.1 B.t.u. internal energy content and 40.6 B.t.u. total heat per pound of air. Next, by following the oblique (entropy) line from 70° on the lower line up to the horizontal line for ratio 6, we find that the temperature at that point, which is that of adiabatic compression, is 424.0° and that at this temperature the heat contents per pound of air are 89.8 and 125.6 B.t.u. The heat equivalent of the work of compression of one pound of air, then, is 89.8 − 29.1 = 60.7 *B.t.u.*, and the net total work, compression plus expulsion minus contra-work, is 125.6 − 40.6 = 85.0 *B.t.u.* For cases in expansion of air the process is reversed, and the initial point is found on whatever line marks the ratio of expansion.

Figure 18 also may be used for finding temperatures as after adiabatic compression or expansion of oxygen, nitrogen, or any gas for which the value for exponent y is 1.400, but the heat content scales are true for air only. It is but a moment's work, however, to compute the heat contents of these gases as at any given temperatures.

Figure 18 enables one to determine quickly the locations of the three governing points, L, J, and H (Figure 14) for air
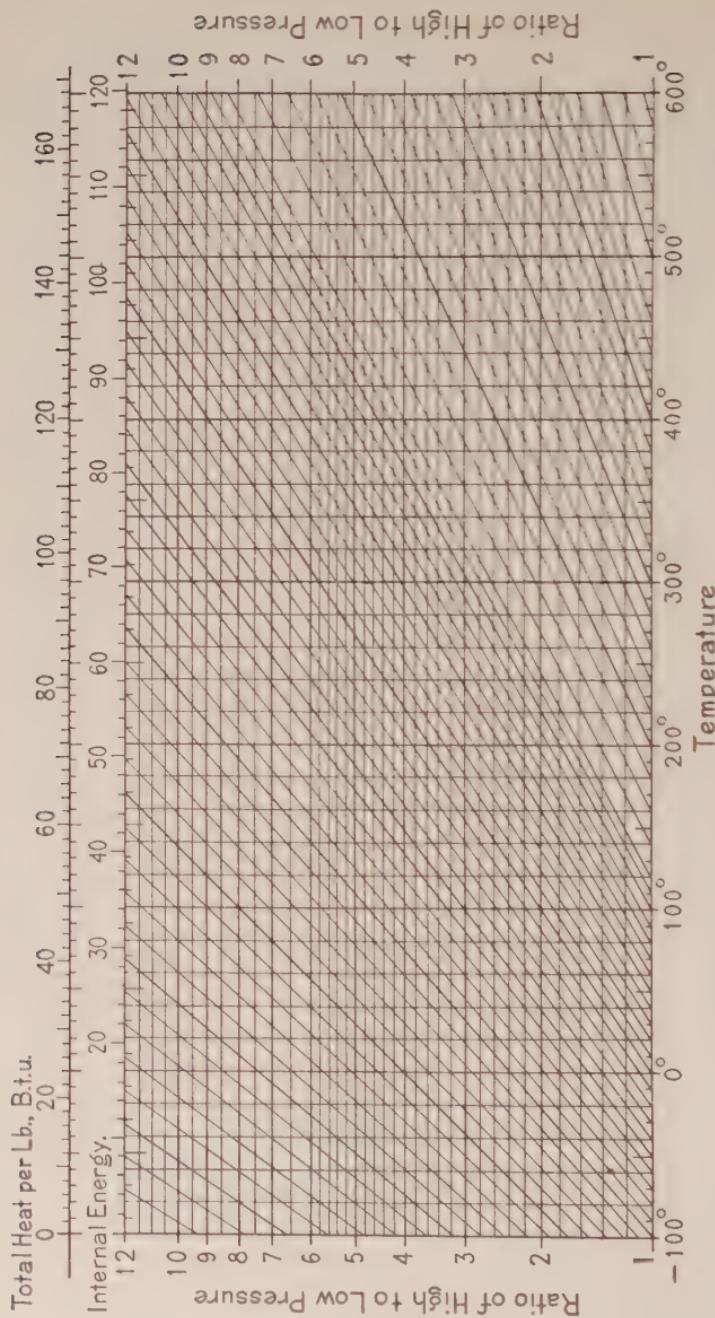
FIG. 18. TOTAL HEAT AND INTERNAL ENERGY CHART FOR AIR

compression or expansion. Point O is fixed by the initial temperature, while P, the fifth governing point, may be located by use of formula (1) or (2) after computation of initial volume.
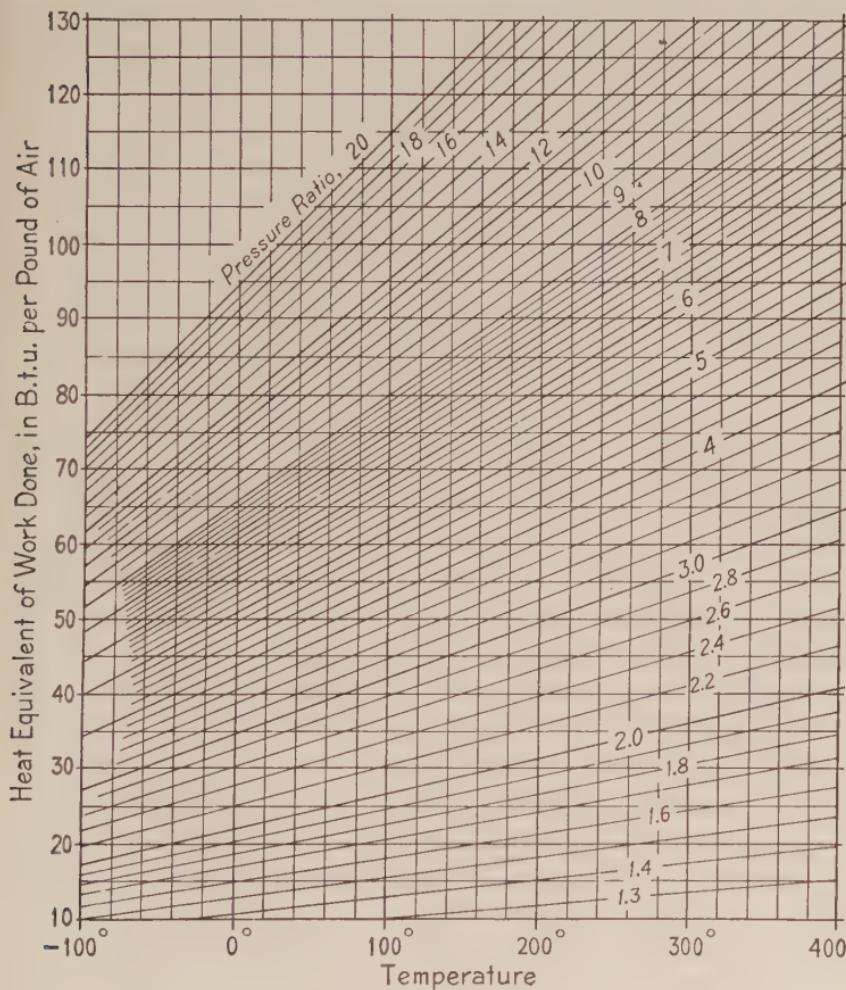
FIG. 19.   HEAT EQUIVALENT OF WORK DONE IN
ISOTHERMAL AIR COMPRESSION

Examining the second of these formulas (page 43) we see that for a given pressure ratio the only variable quantity is T, the absolute temperature, R being constant, and further, that the work varies directly as the absolute temperature. This being

true, it is an easy task to construct a straight-line chart, Figure 19, from which one may read the work of isothermal compression (or expansion). As an example, if the isothermal change in pressure takes place at 100° and the ratio of high to low pressure be 7, the work done on (or by) the air is read directly as the equivalent of 75 B.t.u. of heat. For ordinary problems in air compression one is thus spared the necessity
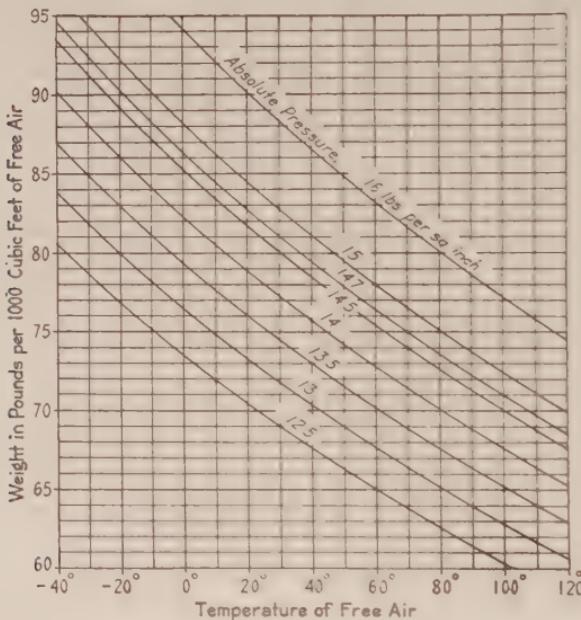
FIG. 20.   WEIGHT OF AIR AT VARIOUS PRESSURES
AND TEMPERATURES

of using formula (1) or (2). Further, this chart is usable for the ideal gases, it being necessary then to multiply the direct reading by the ratio which the value of R for the gas bears to that of R for air. Thus for oxygen multiply the direct reading as for air by $\frac{48.283}{53.294} = 0.906$; for nitrogen multiply the direct reading by $\frac{54.192}{53.294} = 1.016$; and similarly for any other ideal gas.

Rate of Air Compression, per Minute. — The heat-work diagram shows work done per pound of the air, gas, or vapor

being compressed. The ordinary air compressor is rated for capacity according to the number of cubic feet of " free " or outside air it is capable of compressing per minute from a standard atmospheric pressure and temperature to some specified higher pressure. The chart appearing as Figure 20 is convenient for use in problems involving capacity. It shows the weight in

TABLE IV

VOLUME OF ONE POUND OF PURE AIR, IN CUBIC FEET, AT VARIOUS PRESSURES AND TEMPERATURES

Temperature	Absolute Pressures, Pounds per Square Inch							
	16.0	15.0	14.7	14.5	14.0	13.5	13.0	12.5
120°	13.41	14.30	14.60	14.79	15.32	15.89	16.50	17.16
110	13.17	14.05	14.34	14.53	15.05	15.62	16.21	16.86
100	12.94	13.80	14.09	14.28	14.79	15.34	15.93	16.57
90	12.72	13.56	13.84	14.03	14.53	15.07	15.64	16.28
80	12.48	13.32	13.58	13.77	14.27	14.79	15.36	15.97
70	12.25	13.06	13.33	13.52	14.00	14.52	15.08	15.68
60	12.01	12.81	13.08	13.26	13.74	14.24	14.79	15.38
50	11.79	12.57	12.82	13.00	13.47	13.96	14.51	15.09
40	11.55	12.32	12.57	12.75	13.20	13.70	14.22	14.79
30	11.32	12.08	12.32	12.49	12.94	13.42	13.93	14.49
20	11.09	11.82	12.07	12.24	12.68	13.14	13.65	14.19
10	10.86	11.58	11.82	11.99	12.42	12.87	13.37	13.90
0	10.63	11.33	11.57	11.73	12.14	12.60	13.08	13.60
− 10	10.40	11.09	11.32	11.48	11.88	12.32	12.79	13.30
− 20	10.16	10.84	11.07	11.21	11.62	12.05	12.51	13.01
− 30	9.94	10.59	10.81	10.96	11.36	11.78	12.23	12.72
− 40	9.70	10.35	10.56	10.70	11.09	11.50	11.94	12.42

pounds per 1000 cubic feet of free air as at various temperatures and absolute pressures per square inch. For instance, at 80° temperature and 14.4 lbs., the weight of air per 1000 cubic feet is 72 pounds. The net total work per pound of air compressed having been found and the number of cubic feet of free air received by the compressor per minute being known, the horsepower required by the machine is readily found. For example, if the net work per pound, as found from a heat-work diagram,

be 80 B.t.u., heat equivalent, and the compressor is known to be handling 400 cubic feet of free air per minute, from 80° tem-
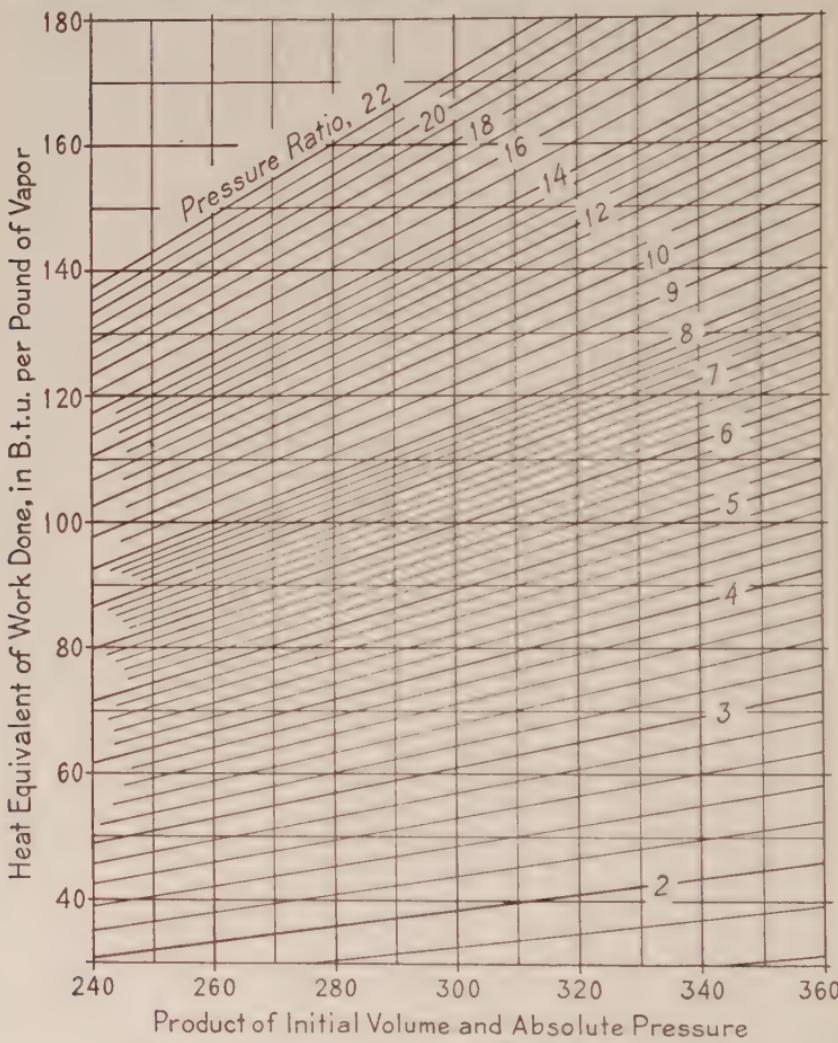
FIG. 21. HEAT EQUIVALENT OF WORK DONE IN
ISO-PV VAPOR COMPRESSION (1)

perature and 14.4 lbs. pressure (absolute), the horse-power rate is

$$\frac{80 \times 72}{42.45} \times \frac{400}{1000} = 54.3 \ H.P.$$

This, it is to be understood, does not include any work necessary for overcoming frictional resistance in any part of the machine,
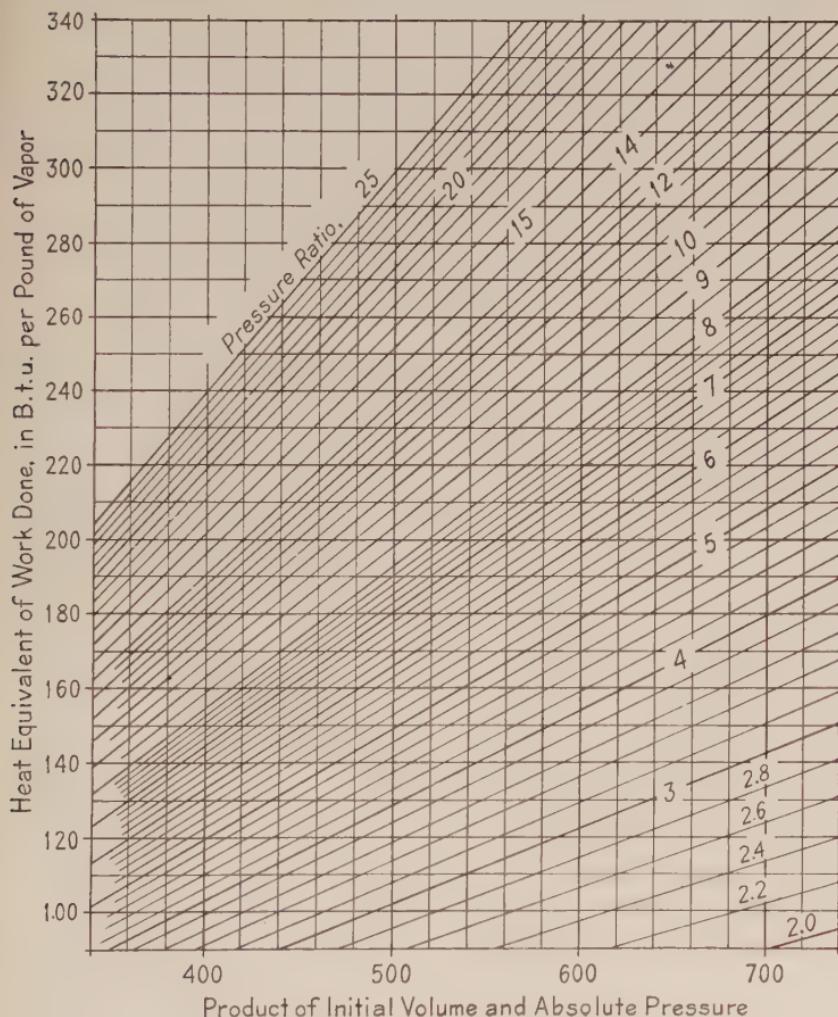
FIG. 22.   HEAT EQUIVALENT OF WORK DONE IN ISO-PV VAPOR
COMPRESSION (2)

to allow for which it may be necessary to add perhaps 10 to 20 per cent. The 54.3 H.P. is work done in reduction of air volume accompanied by rise in pressure. Table IV, giving the volume of air, per pound at various pressures and temperatures,

may be useful in a variety of problems. The volumes listed
are those used as data in constructing Figure 20.

Chart for Work of Iso-PV Compression. — It is possible to
construct charts from which may be read the work of *Iso-PV*
compression of ammonia and other vapors, these charts being
similar in purpose to the chart for air as presented in Figure 19.
Figures 21 and 22 are charts of this character, Figure 21 apply-
ing especially to ammonia vapor as used in refrigerating ma-
chines. To use this, first find in the ammonia tables the volume
of the ammonia vapor per pound as at the given initial pressure
and temperature and multiply it by that pressure in pounds per
square inch, absolute. For normal working conditions, the
product is seldom less than 240 or more than 360, the scope
of the chart. With any *PV* product, say with that correspond-
ing to 20-lb. ammonia at 10° temperature, which is 20×14.44
$= 288.8$, enter the chart from bottom and trace upward along
the vertical which marks this product to its intersection with
the diagonal line for the pressure ratio being used, then to the
left horizontally, and read the work in B.t.u. of heat equivalent
on the vertical scale. As examples, for compression ratio 8, the
work is read as 111.2 B.t.u.; for compression ratio 5, it is 86.5
B.t.u., etc. The chart is equally serviceable for the case of ex-
pansion. A similar chart could be constructed for any vapor the
properties of which are known, and this Figure 21 may be used
for any vapor (or gas) for cases where the initial *PV* product
falls within its scope. For larger products, the similar chart,
Figure 22, may be found useful. Both plot results obtained
by use of formula (1).

CHAPTER IV

EXPANSION OF GASES AND VAPORS

The Mechanics of Expansion. — When a gas or vapor expands against a moving piston and thereby does work, as in the cylinder of a steam or compressed-air engine, its behavior is in accord with the laws and formulas already given for compression. Figure 23 shows the cylinder and valve arrangement of a simple slide-valve engine. Steam enters the cylinder from the steam chest, C, through the inlet port, i, at the pressure existing in the boiler minus whatever losses may be chargeable to friction, valves, pipe-bends, etc. Its pressure is exerted against the face of the piston, P, and moves it to some point, b, at which point the moving slide-valve covers the inlet port and cuts off further inflow of steam. The work done by the steam to this point is the product of its pressure, P_c, multiplied by whatever distance the piston has moved, V_c. It is convenient to take the weight of steam received as one pound and the area of the piston face as one square foot, so that V_c in feet is numerically equal to the volume in cubic feet of the pound of steam at inlet pressure and temperature. The work done by the steam during admission is measured by the rectangle c–b–e–d of sketch (B), corresponding to the work of expulsion from a gas or air compressor. (See Figure 9, for comparison.) No heat energy is spent by the steam during this portion of the piston stroke. The energy used is, in fact, external work done by heat in the boiler, heating water, generating and superheating the steam (see page 23) and transmitted to the piston through the steam supply main by the continuous column of high-pressure steam contained therein.

When the valve has cut off the steam supply, that portion which has entered the cylinder begins to expand and to part with or spend the internal energy that it had acquired in the

boiler and superheater. As it expands, its pressure falls and its temperature likewise will fall unless maintained by a supply of heat coming in through the metal walls of the cylinder to replace that spent by the steam in doing work on the piston. It is theoretically possible thus to supply heat to the extent that the
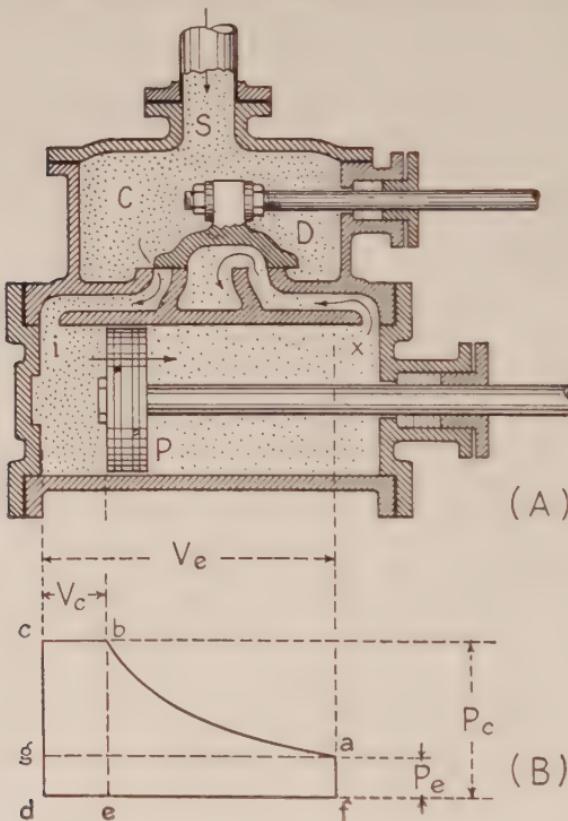
FIG. 23.   TYPICAL CYLINDER AND PRESSURE-VOLUME
DIAGRAM FOR EXPANSION OF GAS OR VAPOR

temperature of the steam at end of the expansion shall be the same as at its beginning, but in practice this is not done. It would be true isothermal expansion. Similarly, in theory, there might be isocaloric expansion, the final and initial heat contents being equal; or an *Iso-PV* expansion to a temperature at which the final pressure × volume product would equal the initial pressure × volume product. (Page 35.)

As the piston continues its stroke, it gradually pushes out the steam that was contained in the cylinder at the end of the preceding stroke of opposite direction, through the exhaust valve, x, and on through the exhaust pipe to the open air, or to a condenser, or to some industrial apparatus in which a portion of its remaining heat content may be utilized. If discharging direct to the atmosphere, the work of exhaust is done against a pressure but slightly above atmospheric, the difference being that due to bends, valves, and frictional resistance; if into a steam condenser, the resisting pressure is lower than atmospheric, perhaps only 3 or 4 lbs. absolute, while if the discharge be into some industrial apparatus the pressure therein is likely to be several pounds above the atmospheric.

Obviously, if the point of cut-off be fixed, so that a constant weight of steam is received at each stroke, and if while doing work the steam is to expand until its final-pressure is exactly that of the exhaust, then the length of stroke and cylinder must be adjustable in order to accommodate varying conditions of expansion, that length not being the same for any two expansion temperatures. Such adjustability cannot be had in an engine; hence the point of cut-off must be made adjustable instead, permitting the weight of steam admitted to vary inversely as the expansion travel.

With any length of stroke, V_e, the work done in pushing out the exhaust steam is equivalent to the product of the pressure, P_e, in pounds per square foot, multiplied by V_e, the length of stroke in feet, the work product being in foot-pounds. The work done by the steam while expanding is that measured by the area under the expansion curve; that is, area b–a–f–e of sketch (B). The net total work is that of admission, c–b–e–d, plus that of expansion, b–a–f–e, minus that required for exhaust, g–a–f–d. This net total is, then, c–b–a–g. This corresponds to the area enclosed by an ordinary indicator diagram, but the latter shows in addition the effects of cushioning, improper valve-setting, etc.

Heat-Work Diagram for Expansion. — Figure 24 is presented to show the construction of the heat-work diagram for a simple

case in expansion. Starting with air at 200° temperature and
88.2 lbs. pressure (per square inch, absolute), it is allowed to
expand down to 14.7 lbs. pressure, working against a piston at
progressively decreasing pressure and to various temperatures,
as at the instant that expansion ceases, the expanded volume,
V_e, being variable. The temperature for adiabatic expansion is
− 64.2°. The final internal energy line marks the internal en-
ergy content of the air as expanded, while the base line, A–B,
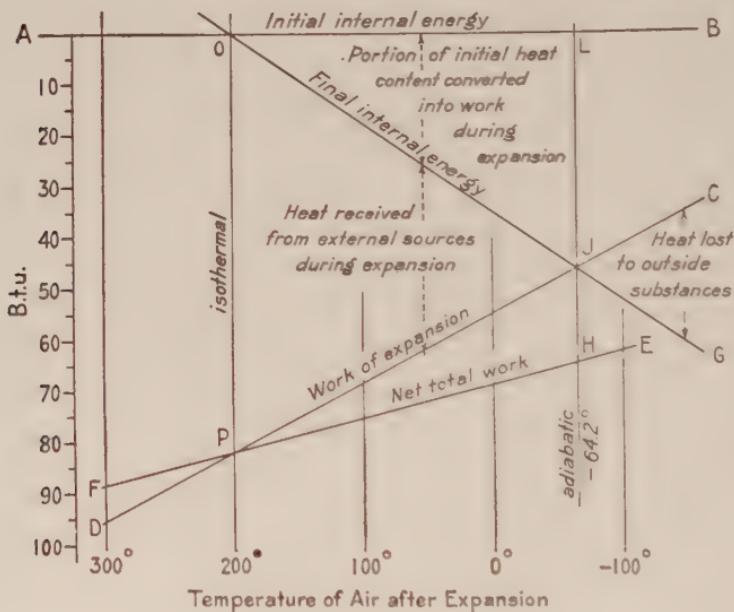
FIG. 24. NORMAL HEAT-WORK DIAGRAM FOR GAS EXPANSION

marks the initial internal energy content. The notation used
is the same as that on the heat-work diagram for compression.
It will be seen that the work accomplished increases with in-
crease in final temperature, but that the work done by the heat
energy of the air itself is greatest at the adiabatic point; to the
left of that point the greater work accomplished must be
credited to heat received from external sources, while to the
right any heat lost to outside substances serves no useful
purpose.

General Graph for Expansion. — Let us again use the case in air expansion presented in preceding paragraph, but with expansion to all temperatures, both practicable and speculative, as was done for the general compression graph (page 56). It is necessary to compute the volumes when expanded, the work done in the expansion, and the net total work by use of formulas
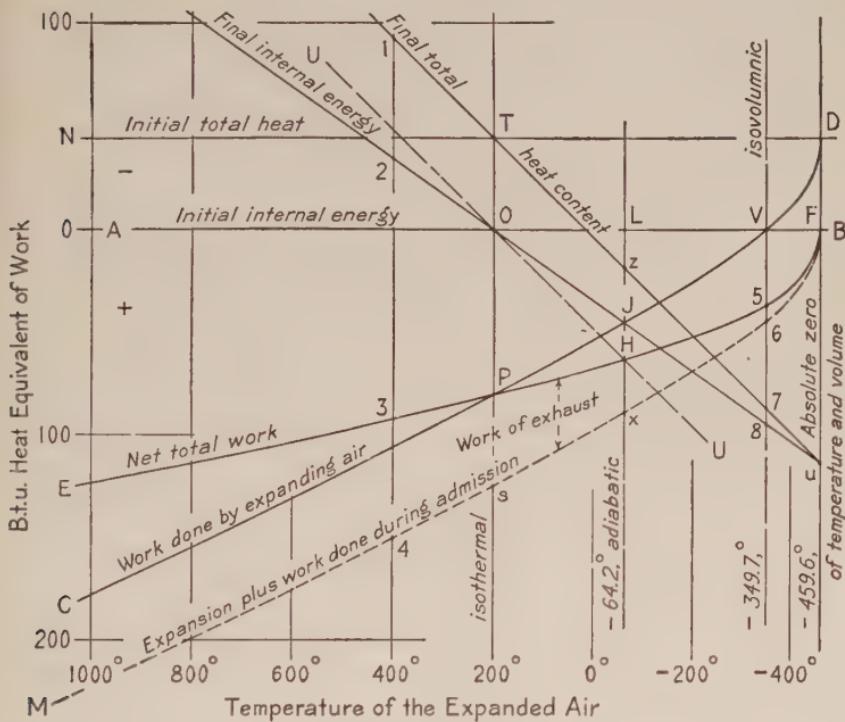
FIG. 25.   THE GENERAL GRAPH FOR GAS EXPANSION

(15), (13), and (14). Having done this for a series of temperatures at reasonably close intervals, we plot the two work curves, E–P–H–F and C–P–J–D, Figure 25. The admission work done by the high-pressure air prior to cut-off is a constant quantity and may be shown on the diagram at a constant distance below the expansion work curve, as curve M–s–x–F. Its numerical value in this case is 45.2 B.t.u. The net total work done at any temperature of exhaust, is the sum of expansion

work and admission work, minus that required for exhaust. At the isothermal point, work of admission and of exhaust are equal; and the net total work is neither more nor less than the work of expansion. At temperatures lower than the isothermal, the net total work accomplished exceeds that done by the air expansion alone, while at temperatures above the isothermal it is less.

If there be no cut-off, the air acting against the piston at undiminished pressure throughout its travel, then the work of expansion is 0, the expansion work curve crossing the initial heat or base line at point V, the isovolumnic point for expansion. In this imaginary case the length of stroke is reduced until it equals the admission volume: that is, the stroke length is V_c, equal to V_e. To accomplish this would require that there should be a sudden cooling of the air at the instant admission ceased, with drop in pressure to one-sixth of the original, but with no change in volume. The computed temperature for this point is $-349.7°$ (one-sixth of the initial absolute temperature). In this discussion we ignore the fact that condensation to liquid air would occur at some point in the very low temperature zone, and assume that air obeys Charles's law down to the absolute zero of temperature.

If the temperature of the expanded air be made lower than that at the isovolumnic point, the volume must become less than the initial V_c, meaning that the piston must follow the air backwards instead of being pushed forward by it as in normal expansion, the result being negative work; that is, work is done on the air instead of by the air. Thus the expansion work curve crosses the base line; and as extremely low temperatures are reached, it turns upward sharply and ends at point D on the absolute zero temperature vertical, where the negative work is equivalent to that of pushing the high-pressure air out of a cylinder of length V_c and is the same numerically as the work done by the air during admission, but opposite in sign. The work of exhaust, measurable at any temperature as the distance between the net total work curve and curve $M-s-x-F$, becomes 0.0 at absolute zero temperature, as at that limit the

air volume is 0.0 and no work is required or possible for its expulsion. The volume at that point being 0.0, pressure per unit of area has no meaning and may be anything between 0 and infinity. Whatever the initial and final pressures may be, the net total work curve and the admission-plus-expansion curve must meet at a point on the absolute zero temperature vertical. Passing now to the high temperature region, both the work done by expansion and the net total work increase with rising final temperature, the limit for each being infinite work at an infinitely high temperature.

The final internal energy curve (straight line in this instance) passes through the isocaloric point, O, (in this case it is also the isothermal point), through J, which is at the intersection of the expansion work curve and the adiabatic temperature vertical, and terminates at a point, u, marking 0 heat content on the absolute zero temperature line. The final total heat line likewise terminates at the point u. Computing the initial total heat content of the pound of air and laying it off the proper distance above the point of origin, u, we fix the position of the horizontal line N–T–D. It intersects the final total heat line at the isothermal temperature point, T, and passes through the point D previously located. Compare this with Figure 10 where the same notation is used, as far as possible. If a straight line parallel to the final total heat line be drawn through point O, it will pass through point H. The following equations are true:

$$J{-}x = D{-}F = T{-}O = H{-}z = P{-}s = \frac{144 \times P_c \times V_c}{777.5}.$$

At any final temperature, the difference between internal energy and total heat is equivalent to the work required for exhaust of the expanded air. Thus at absolute zero this work is 0.0; at the isovolumnic point it is $7{-}8 = 5{-}6$; at the adiabatic, it is $J{-}z = H{-}x$; at the isothermal, it is $T{-}O = P{-}s$; at 400° it is $1{-}2 = 3{-}4$; etc.

If we plot the values of exponent n as found and used for the series of computations which control the work curves of

Figure 25, we trace the curves shown in Figure 26. In this, at the isothermal point, $n = 1.00$; at the adiabatic, $n = y = 1.40$; from that point to the isovolumnic, it becomes larger and larger, sharply, until it is infinitely great at the isovolumnic temperature; between that and absolute zero, the values are negative, varying from infinity down to 0; in the other direction, at temperatures above the isothermal the values of n,
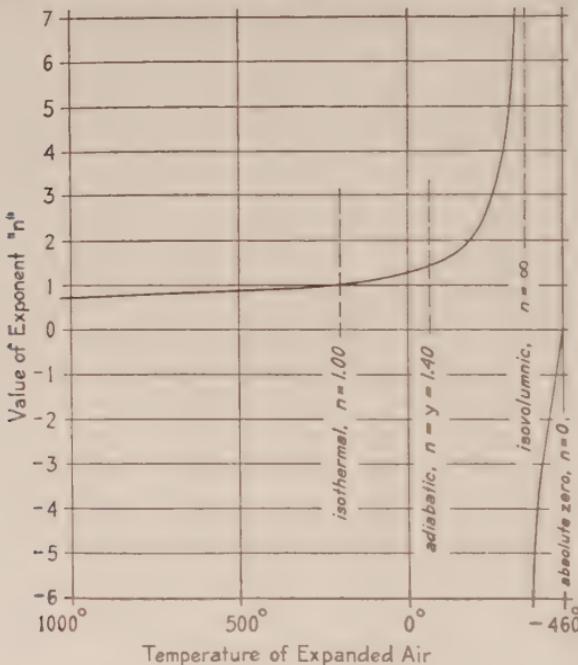
FIG. 26.   PLOTTED VALUES OF EXPONENT N FOR
TYPICAL CASE OF EXPANSION

while positive, are less than 1.00, diminishing but slowly with rising final temperature and finally becoming 0.0 at temperature infinity. Compare with Figure 11 for compression.

In order to compare the adiabatic, isothermal, and other expansion curves for pressure and volume, we construct Figure 27, comparable with Figure 12, for the case of compression. Here the initial volume, V_c, is constant and the expanded volume, V_e, is variable. Of course, in an actual engine this is not possible, and the point of cut-off and consequent volume

of air or steam admitted at full pressure are varied to suit the
pressure and heat conditions, but for this discussion we assume
that the expanded volume may vary as it will. Using air with
initial pressure 88.2 lbs. and 200° initial temperature and ex-
panding it down to 14.7 lbs. pressure, we know that with
isothermal expansion, for which $n = 1.00$, the expanded volume
is six times the initial volume. For adiabatic expansion the
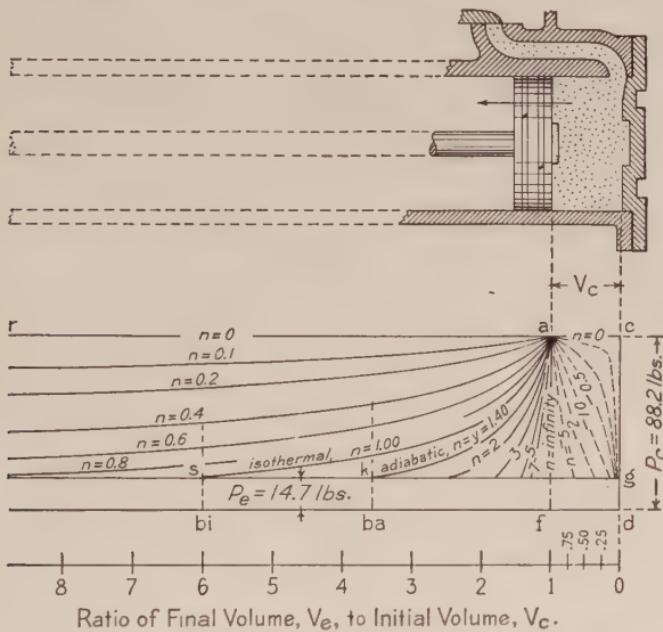
FIG. 27.   PRESSURE-VOLUME RELATIONS IN EXPANSION

final volume ratio is found by dividing 1.00 by the factor 0.2781
from Table II, the quotient being 3.595; that is, the volume
when expanded adiabatically is 3.595 times the initial volume.
The greater the value of n the smaller the volume ratio, until
finally at $n =$ infinity, the isovolumnic point, the volume ratio
becomes 1.00 and there is no change in volume. For tempera-
tures above the isothermal, where n is less than 1.00, the
expansion curves straighten gradually as shown until, at tem-
perature infinity, the curve is identical with the straight line
$a–r$, and here $n = 0.00$. Expansion curves for the negative-

work part of the diagram take the positions shown, the values of n, all negative, varying from infinity to 0.00.

The work done by expansion is measured by the area under the expansion curve. For the adiabatic condition it is the area $a-k-ba-f$; for the isothermal it is $a-s-bi-f$; at the iso-volumnic point there is no area under the vertical line and no work; at absolute zero temperature, where both volume and n become 0.00, the expansion curve coincides with lines $a-c-g$, and the work area under this " curve " is $a-c-d-f$, negative, numerically equal to the work of admission and the equivalent of the difference between initial total heat and internal energy contents.

CHAPTER V

PRESSURE CHANGES, GENERAL CASE

Ratios of Pressure, Temperature, and Exponent n. — Formula (15) for the exponent n as applied to polytropic changes in pressure involves pressure and volume ratios, and may be written in this form:

$$n = \frac{\log\left(\dfrac{P_c}{P_e}\right)}{\log\left(\dfrac{V_c}{V_c}\right)}. \tag{16}$$

It is possible to construct a formula for the value of n which shall involve ratios of absolute temperatures instead of volumes. Starting with the basic equation, $PV = RT$, and letting all symbols used have same meaning as in preceding equations, we have

$$P_c V_c = RT_c, \text{ and } P_e V_e = RT_e, \text{ or } P_e = \frac{RT_e}{V_e}.$$

For convenience, let P_c/P_e be some concrete number, say 5 then

$$P_c = 5P_e, \text{ and } V_c = \frac{RT_c}{P_c} = \frac{RT_c}{5P_e} = \frac{V_e T_c}{5 T_e}.$$

Substituting this value for V_c in the denominator term of (16), it follows that

$$n = \frac{\log 5}{\log 5 + \log\left(\dfrac{T_e}{T_c}\right)},$$

and now replacing the concrete by the general ratio, we have

$$n = \frac{\log\left(\dfrac{P_c}{P_e}\right)}{\log\left(\dfrac{P_c}{P_e}\right) + \log\left(\dfrac{T_c}{T_c}\right)},$$

which is equivalent to

$$n = \frac{\log\left(\dfrac{P_c}{P_e}\right)}{\log\left(\dfrac{P_c}{P_e}\right) - \log\left(\dfrac{T_c}{T_e}\right)}. \tag{17}$$

In this, it will be observed, the symbols for volume have disappeared, and we have to deal with ratios of pressure and of absolute temperature.

As an example in use of formula (17), let us take the same case in oxygen compression as used on page 56, where a value for
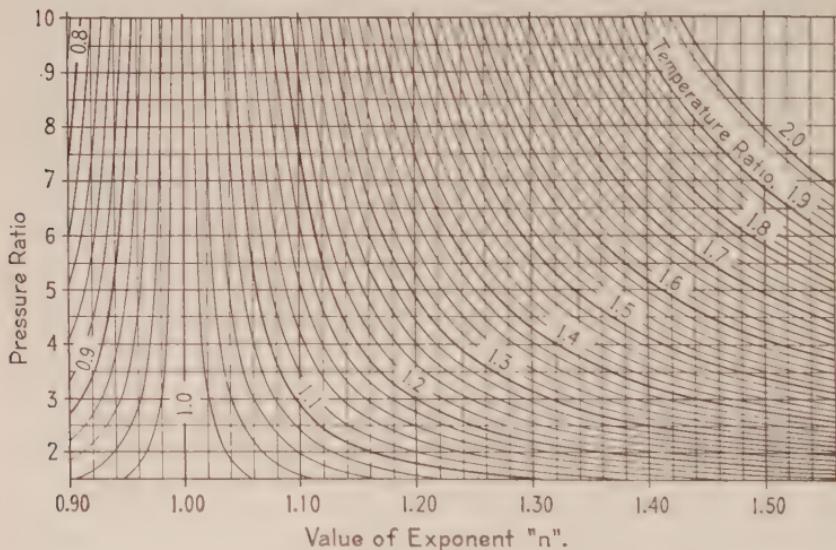
FIG. 28.   CHART FOR FINDING VALUE OF EXPONENT N,
COMPRESSION OR EXPANSION

n was found by formula (15). The pressure ratio is 4.5, the logarithm of which is 0.6532. The ratio of the absolute temperatures is $700 / 530 = 1.3208$, the logarithm of which is 0.1208. Then substituting these in formula (17), we have,

$$n = \frac{.6532}{.6532 - .1208} = \frac{.6532}{.5324} = 1.226,$$

as found on page 56.

This value for n, it should be observed, is true for air or for

any perfect gas, for the ratios of pressures and of absolute temperatures used in the problem.

By formula (17), it is possible to construct a chart showing values of the exponent n for the normal ranges of pressures and temperatures used in ordinary compression and expansion of air and gases. Figure 28 is such a chart. Using this to solve the problem of preceding paragraph, we enter with pressure ratio 4.5, follow that horizontal line to intersection with temperature ratio 1.3208, trace from that point a vertical line to the bottom scale, and there read the value for n as 1.226. The accuracy of the readings, of course, cannot equal that of computation by formula (17) or (15), but the chart is a useful tool in the solution of ordinary problems, being especially convenient in finding n for use in formulas (13) and (14). It is more convenient, generally, to substitute for the constant R used in those formulas its equivalent value, explained on page 15,

$$R = (C_p - C_v) \times 777.5.$$

Using this, formula (14) may be expressed thus, for the general polytropic case:

$$Net\ work\ in\ foot\text{-}pounds = \frac{n(C_p - C_v)\ 777.5\ (T_c - T_e)}{n - 1},$$

and, as we desire to obtain the work in B.t.u. of heat equivalent, we divide the work in foot-pounds by 777.5, and then have

$$Net\ work,\ in\ B.t.u. = n(C_p - C_v)\frac{(T_c - T_e)}{n - 1}. \qquad (18)$$

Values for $(C_p - C_v)$ may be found in Table I.

The following examples show the utility of Figure 28, used in conjunction with formula (18).

(a). Air is compressed from initial temperature 50° to final temperature 300°, the initial pressure being 14.7 lbs. and the final pressure 88.2 lbs. per square inch (all pressures absolute). What net total work is required per pound of air? To solve this, we have the pressure ratio, which is 6, and the ratio of the absolute temperatures, $760/510 = 1.49$. Then substituting the several proper values in formula (18), we have

$$Net\ total\ work = 1.286 \times 0.0685 \times \frac{(300 - 50)}{0.286} = 76.8\ B.t.u.$$

(The value of n, 1.286, being found by use of Figure 28.)

(**b**). Helium gas is compressed from initial 15 lbs. and 60° up to final 90 lbs. and 225°. What is the net total work per pound of the gas compressed?

The value for n in this case is found to be 1.182. Then

$$Net\ total\ work = 1.182 \times 0.497 \times \frac{(225 - 60)}{0.182} = 532.6\ B.t.u.$$

(**c**). In one of the cylinders of a multi-stage compressor, nitrogen is compressed from initial 52 lbs. to final 195 lbs., the initial temperature in cylinder being 108° and that of discharge 219°. What work is done per pound of nitrogen? Here the value for n is read as 1.156, and we have

$$Net\ total\ work = 1.156 \times \frac{.0698 \times 111}{.156} = 57.4\ B.t.u.$$

(**d**). An air engine uses air at 131 lbs. pressure and 143° temperature, expanding it down to 15 lbs. and 37° temperature. What work is done per pound of air? Here, n being read as 1.10, we have

$$Net\ total\ work = 1.10 \times \frac{.0685 \times 106}{.10} = 79.9\ B.t.u.$$

(**e**). Using the same data as problem (*a*), what would be the temperature after adiabatic compression? In that case, $y = n = 1.40$, and the intersection of the lines marking pressure ratio 6 and $n = 1.40$ is at temperature ratio 1.668, which, multiplied by the initial absolute temperature, 509.6°, gives the absolute temperature as after adiabatic compression, 850.0° or 390.4° ordinary. It will be observed that the temperature ratio 1.668 is identical, as it should be, with that given in Table III.

General Graph for Pressure Changes. — If curves similar to those of Figures 10 and 25 be made for any given pressure ratio, for air or for any perfect gas, it will be seen that they have cer-

tain elements in common and that they may be combined to form one general graph, as in Figure 29, which is for air, the initial temperature being 150° and the ratio of pressures being 5. The curves, plotted as the results obtained by use of formula (18) and the corresponding formula for work of compression or
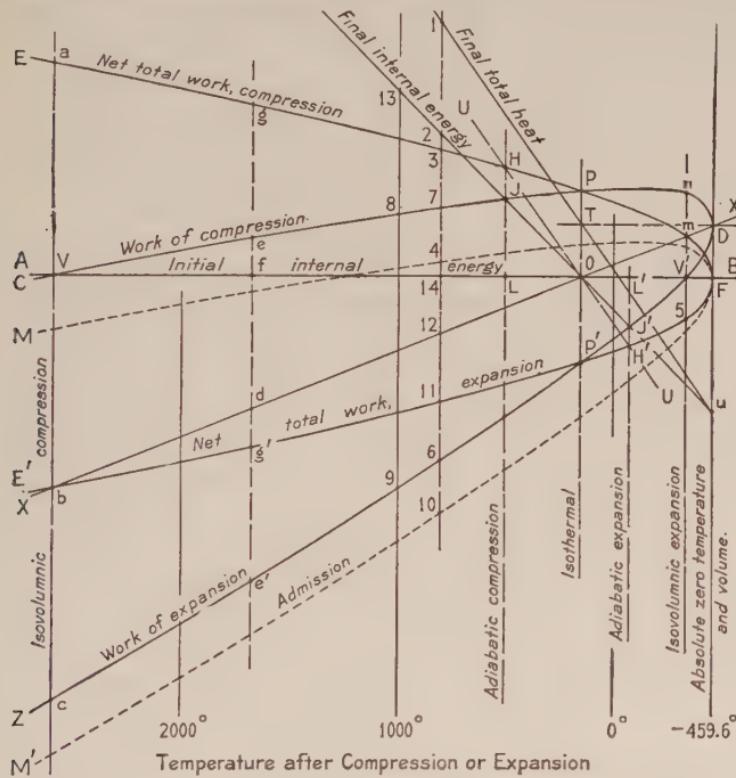
FIG. 29.   THE GENERAL GRAPH FOR COMPRESSION AND
EXPANSION

expansion, serve for any case where the pressure ratio is 5. The truth of this statement may be shown as follows, omitting mention of formula (18).

The location of point P, considering at first only the compression curves, is fixed by formula (1) or (2), in which PV is a constant quantity regardless of the actual pressures, permitting the use of Figure 19, which takes no account of pressures other than the ratio between the initial and the final. Points J and H

are likewise fixed, both as to temperature and work in B.t.u., by Chart 18 or the formulas used in its construction; and for any temperature change the results are identical, starting with any pressure and ending with pressure changed in any specified ratio. That is, starting with temperature 150° and pressure 75 lbs., and compressing at ratio 5 to 375 lbs., we find that the final temperature is the same as if starting from 20 lbs. and ending with 100 lbs., the multiplier from Table III being the same in each instance or for any other with pressure ratio 5. The work done, using formulas (13) and (14), varies only with temperature change, which is constant for any pressure ratio and initial temperature; hence in these cases the work is constant.

This reasoning applies to all polytropic conditions and hence to all points on the curves, both for compression and expansion. Point D, the distance between which and point F measures the work done by the air during admission, for either compression or expansion, is determined by the initial temperature and that only, being independent of pressures and of pressure ratio as well. Distance D–F is equal to T–O, which is the difference between total heat content and internal energy of the air at the initial temperature, measured above 0.0 heat contents at absolute zero temperature. The isovolumnic point for compression, V, is dependent for location on pressure ratio, distance V–F being equal to temperature O–F multiplied by that ratio. In this case we have,

$609.6° \times 5 = 3048°$ *absolute, or* $2588°$ *ordinary temperature.*

Similarly, point V' for expansion is located at one-fifth of O–F, measuring from F, being independent of the actual pressures but governed by their ratio. Thus the work as at any final temperature, as measured to either of the curves, is in fact determined by the initial temperature and the ratio of pressures.

The two compression curves, C–P–D and E–P–F, are drawn as explained on page 57, and the two expansion curves, Z–P'–D and E'–P'–F, as explained on page 83. It is at once apparent that the two branches of the curve for net total work are op-

posed duplicates of each other, the net work at any final tempera-
ture being exactly the same for expansion as for compression.
For instance, at 1000° final temperature the calculated work of
compression is 47.51 B.t.u., the value of n is 2.2261, and the net
total work is 105.8 B.t.u., while the calculated work done in ex-
pansion to this same temperature is 164.1 B.t.u., the value of n
is 0.6449, and the net total work done is 105.8 B.t.u. per lb., the
same as for compression. It may be remarked here that in both
cases the specified temperature of discharge could be attained
only with the aid of a considerable quantity of heat from ex-
ternal sources of supply. In compression the heat from external
sources would be that measured by distance 13–8, being about
97 B.t.u. per pound of air compressed; and in expansion it would
be measured by distance 13–9, being about 309 B.t.u. per lb. of
air.

If the distance between the compression curve $C–P–D$, and
the expansion curve, $Z–P'–D$, be bisected, at any temperatures,
the points thus determined will lie on a straight line, $X–X$,
passing through points O and D. Furthermore, this line or axis
$X–X$ intersects the isovolumnic compression temperature ver-
tical at the same point as the expansion net total work curve, as
at point b. It follows that $a–V$, $V–b$, and $b–c$ are equal work
quantities. It is also apparent that at this point, where n for
compression has the value infinity, n for expansion has the value
0.5. Similarly, at the isovolumnic expansion temperature,
marked by V', point m is at the intersection of that temperature
vertical, axis $X–X$, and the net total work of compression
curve. Quantities $n–m$, $m–V'$, and $V'–5$ are equal, and n for
the compression side has the value 0.5, that for expansion being
infinity. These complementary values for n, 0.5 and infinity,
will be found true with any pressure ratio and any initial tem-
perature. Moreover, in any case, with any pressure ratio, which
we will designate by the symbol r, the vertical distance from the
isovolumnic point to the axis $X–X$, as at $b–V$ and $m–V'$, is
equal to $D–F$ $(r-1)$ for the compression isovolumnic, and to

$$D-F\left(\frac{r-1}{r}\right)$$

for the expansion isovolumnic. In the case plotted as Figure 29, $b-V$ is 4 times $D-F$, and $m-V'$ is four-fifths of $D-F$, all measurements being in B.t.u.

Again, at any discharge temperature the work measured vertically between axis $X-X$ and the compression or the expansion curve, such as at $d-e$ or $d-e'$, is the exact equivalent of the net total work as measured from the initial heat axis, $A-B$, to points g and g'. It follows that $e-f$ is equal to $d-g'$, also that $g-e$ is equal to $g'-e'$ and to $f-d$. The curve $M-F-M'$, marking the work done by the air during admission to the cylinder, is at uniform distance $D-F$ below the compression-expansion curve. Being thus fixed by $D-F$, that distance is dependent solely on initial temperature and independent of pressure and of pressure ratio.

As previously stated, the value of n at any temperature is findable by dividing the net total work by the work of compression, or by the work of expansion if expansion is being considered. For instance, at $800°$ n for compression is the quotient of the work measured by $3-14$ divided by that measured by $7-14$, while the n for expansion to same temperature is $14-11$ divided by $14-6$. It is a matter of interest that the two values of n, at any temperature, are inversely proportional to the work of compression and expansion. Thus, at $1000°$ the values of n were found (page 95) to be 2.2261 and 0.6449, and the work there was 47.51 B.t.u. for compression and 164.1 B.t.u. for expansion. Here we have the relation

$$\frac{2.2261}{.6449} = \frac{164.1}{47.51}.$$

Thus it is seen that in the general case we have two complete curves, each with two branches, one branch for the compression region and the other for expansion; that at any temperature, points on the two branches of either curve are equidistant from an axis, the particular axis being $A-B$ for the branches of the net total work curve and $X-X$ for the compression-expansion work curve; and that the position of axis $X-X$ is determined by the initial temperature. It follows that, if we can plot any one of the four curve branches, we can therefrom scale

all measurements necessary to plot the other three branches. Moreover, no matter what one of the four be desired, we can locate five points thereon, and this without use of any formula or chart for finding any value for exponent n; that is, we can locate the isothermal point, P or P', and the adiabatic J or J', or else H or H'; point F is constant, point D is easily located by calculation, its height in B.t.u. above point F being the product of the initial absolute temperature multiplied by $(C_p - C_v)$ for the gas; point b is likewise readily found, determining points a and c, or point m determining points n and 5. Through the five points thus located, one can draw the curve with a fair degree of accuracy and follow with the other three branches if desired.

The complete curve, $E-P-F-P'-E'$, considered as a whole, quite closely resembles a parabola, but is not a parabola. The other curve, $C-P-D-P'-Z$, likewise is a near approach to an oblique parabola, its axis, $X-X$, being inclined to that of the normal "parabola" (so-called for the moment) crossing it at O, the included angle made by the two axes being $D-O-F$, this angle being a constant for air or for any perfect gas. The work measured by $D-F$, in B.t.u. of heat equivalent, is that done by the air or gas during its admission to the cylinder of the compressor or engine. The work done in expulsion of the air from the machine, whether it be the compressed air from a compressor or the exhaust from an engine, is measured at any temperature by the vertical distance between the admission curve, $M-F-M'$, and the net total work curve. For instance, at 800° it is 3–4 for compression or 11–10 for expansion, either being equivalent to 1–2, which is the difference between the final total heat and internal energy contents of the air (one pound) at that temperature and at any pressure. As previously pointed out, adiabatic points J and J' fall on the final internal energy line, while points H and H' fall on the line $U-U'$ drawn through O parallel to the final total heat line.

Inspection of Figure 29 shows that, starting at a given temperature and pressure, adiabatic compression to some higher pressure requires the expenditure of more power than is re-

turned in case the gas or air be expanded at the same pressure ratio. That is, work $H-L$ is materially greater than work $H'-L'$. If, however, the air be compressed adiabatically to the higher pressure and then allowed to expand adiabatically
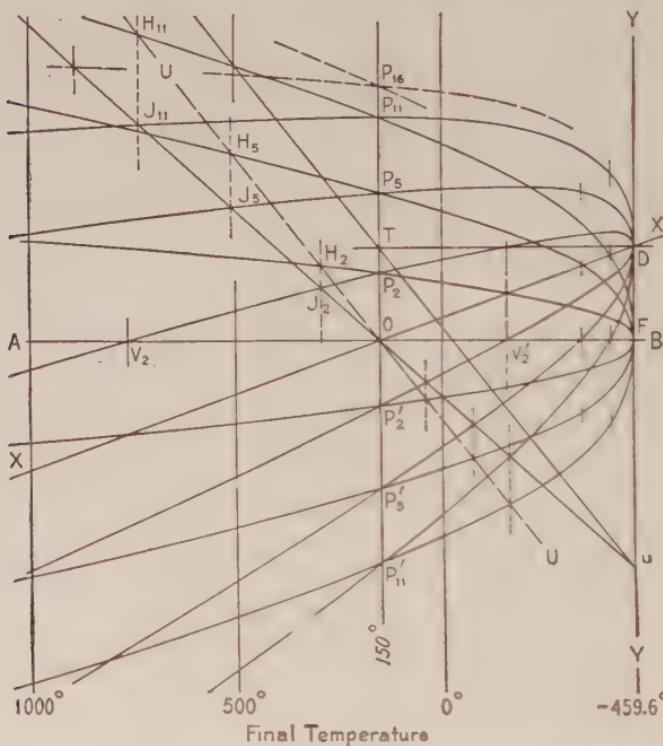
FIG. 30. INFLUENCE OF PRESSURE RATIO ON WORK
CURVES

from that higher pressure back to the original, then the work returned is equal to that expended, as the ratio of temperatures is equal for the two cases.

In Figure 30 are shown three sets of work curves for air compression and expansion, using pressure ratios 2, 5, and 11, the initial temperature for all being 150°. The point P rises with increase in pressure ratio, infinity being its upper limit, and falls with decrease in ratio to O as the lower limit. The vertex of all net total work curves is at F on the initial heat

base line, while that of all compression-expansion work curves
is at D on the oblique axis $X-X$. For pressure ratio 0.0 each
branch of the net total work curve coincides with axis $A-B$,
and each branch of the compression-expansion work curve
coincides with axis $X-X$. With increasing pressure ratios,
the curves become more and more obtuse until at ratio infinity
each coincides with the vertical axis $Y-Y$. It is evident that
in the case of compression there is no pressure ratio at which
the net total work at the isothermal exceeds that at the adia-
batic temperature, because for any ratio greater than 0, the
net total work curve ascends towards the left. For expansion,
on the other hand, the net total work at the isothermal in all
cases exceeds that at the adiabatic, the curve ascending to the
right towards that point. The work of compression only, how-
ever, at the adiabatic point may be less than, equal to, or
greater than the work at the isothermal. It is seen to be less
at ratios 2 and 5, about equal at ratio 11, and greater at ratio
16, for which only a segment of the work curve is shown. In
expansion, on the other hand, the work done by the expanding
gas is always less at the adiabatic point than the net total
work, and the latter has been shown to be less than at the
isothermal. However, as stated on page 82, the greater work
rendered by the air expanding isothermally is to be credited
to heat received from some outside source. For a steam or air
engine there is no outside source of importance, and the work
done is that due to expenditure of internal energy plus such
small gains as there may be from the metal of the engine, the
surrounding air, or, in some machines, from a steam jacket.

Cylinder Heating Compared with Pre-Heating. — When air
or steam is expanded in the cylinder of an engine, doing work,
for any given value of exponent n the work done increases with
increase in the initial temperature. Formula (18) shows the
truth of this statement, as it shows that the work varies with
the difference between T_c and T_e; and for a given ratio of
T_c/T_e their difference increases with T_c. Any heat expended
in pre-heating, therefore, effects an increase in the work done.
Now the expansion graphs, such as Figures 24 or 29, show that

heat given to the air or steam during its expansive stroke like-
wise increases the net total work done as compared with that
done by adiabatic expansion. The question arises as to which
of these uses of a given quantity of heat yields the greater
gain. A simple heat-work diagram, Figure 31, answers the
question.

Air at 120° initial temperature is expanded in an air engine
with pressure reduction ratio 6. If the expansion could be
adiabatic, the final temperature would be − 112.1°, and the
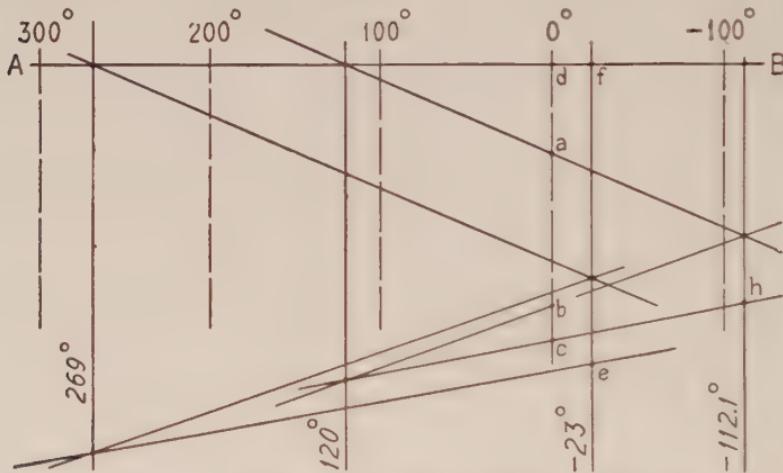
FIG. 31.   EFFECT OF CYLINDER HEATING COMPARED
WITH PRE-HEATING OF GAS IN EXPANSION

net total work 55.6 B.t.u. per lb. of air. In actual practice
this low temperature would not be attained, because of the
ingress of heat from without, such as from the surrounding air,
metal, etc., nor would it be practicable to work at such low
temperature on account of frosting and similar troubles. Let
us assume that the final temperature is held at 0°, with the aid
of a steam jacket or some similar source of heat having a
definite money cost. Then the heat expended in heating each
pound of air in the cylinder is that measured by $a–b$, which
scales 35.7 B.t.u., and the net total work done is $d–c$, which is
64.5 B.t.u. Now, for comparison, let the 35.7 B.t.u. be used
otherwise to heat the air before it comes to the expansion cylin-

der, raising its temperature from 120° up to 269°, (35.7/0.2396 = 149° rise), and let this warmer air expand adiabatically down to temperature − 23°. Now the net total work is that measured by *f–e*, being 69.9 B.t.u., which exceeds by 5.4 B.t.u. the work gain for the expenditure of a like quantity of heat in cylinder heating. As stated, however, cylinder heating may be necessary to prevent troubles due to excessively low temperatures of the exhaust or to condensation.

CHAPTER VI

PISTON FRICTION

Heat of Friction. — Mention has been made of the fact that there is always generation of heat by friction of the piston against the walls of a compressor or engine cylinder, and the influence of this heat will now be considered; the case of compression first. The curve for net total work of compression, in a heat-work diagram, measures the work actually done in reduction of volume of the gas or air being compressed and in expelling it from the cylinder, less the assistance given by the incoming low-pressure gas or air. In addition to this a certain amount of power must be spent in overcoming the resistance due to friction, this additional mechanical energy being transformed into heat and raising the temperature of the metal parts that are in sliding contact. A part of the heat thus temporarily lodged in the metal passes to the cold water circulating through the water-jacket, part to the air of the machine room by radiation, part to the low-pressure air or gas which is following the piston, and a part to the gas or air being compressed. Some other parts of the machine, such as shaft bearings, cross-head guides, piston-rod packing, crank pins, and minor moving parts, also generate heat of friction. The total energy spent in overcoming friction in the machine may be determined by test in any given case, measuring the power actually applied to the compressor and comparing it with the power expended on the gas or air as shown by an indicator card or by a heat-work diagram. It might be found that machine friction added 10 to 20 per cent to the power which would be required if there were no friction. It is hardly practicable to find exactly what portion of this added effort is spent at the piston rings and rod packings, but probably it is not less than one-half; and if a heat-work diagram should

show the net total work for a given case to be, say, 120 B.t.u., we might safely estimate that an additional 8 to 12 B.t.u of energy would be spent in overcoming piston friction and rod-packing friction. Of course, the amount varies with the surface condition of the parts, efficiency of lubrication, and the working pressure against which the piston must be tight.

Figure 14 shows how any heat acquired from external sources in the course of the compressive stroke raises the temperature of the gas, decreases the work of compression, and increases the net total work required. Without any such gain and with no loss, the final temperature in that case would be 390.4°, the adiabatic; with quantity v–w gained and none lost, the final temperature would be 435°; with none gained and quantity a–b lost, it would be 300°. If quantity v–w be gained and the final temperature be 300°, the heat carried away by jacket water, etc., is not a–b only, but a–b + v–w. Any frictional heat generated during the compressive part of the stroke is heat received from external source, exactly as v–w, and is the heat equivalent of the extra work required. That portion of it which is carried away by jacket water does not influence the final gas temperature or the work actually done on the gas, but requires an expenditure of equivalent power. It also affects the weight of jacket water required, or its temperature rise in the jacket chambers. Some portion of the heat of friction passes to the cool, incoming low-pressure gas and is one of the several items which determine the initial temperature.

Figure 32 is, in part, a straight-line heat-work diagram for a typical case in compression. In this, v–w represents the heat equivalent of the work required to overcome friction of the piston in the compressive part of the stroke when the final temperature is t_1. The compression stroke is shorter at temperature t_1 than at the adiabatic and considerably shorter than at the isothermal; hence if the heat of friction with final temperature t_1 be v–w, at the adiabatic it will be slightly more than v–w, proportional to the greater distance the piston moves, while at the isothermal it will be still more, in proportion to the length of the isothermal stroke. Thus it is possible to draw

the line s–s, the vertical distance between which and line C–D, at any final temperature, measures the work spent in overcoming frictional resistance during compression. At temperature t_4, for instance, it is 1–2, and the heat carried away by jacket water, etc., is 2–3; whereas if there were no frictional heat, it would be only 1–3. If the jacket water can receive only
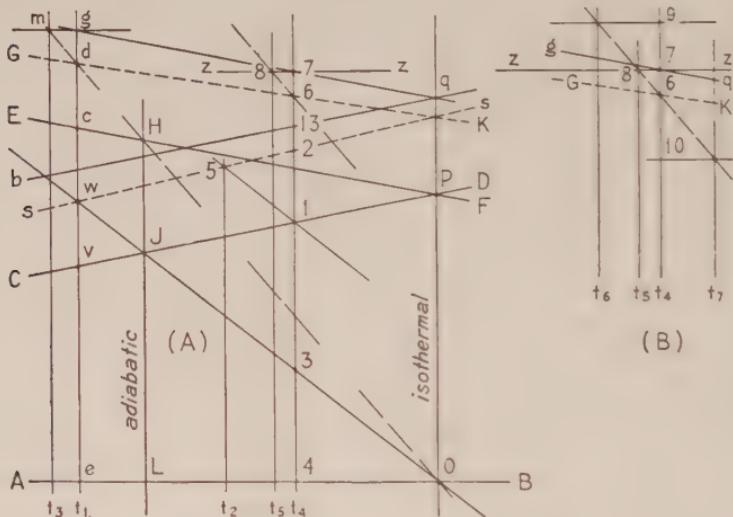
FIG. 32.　DISTRIBUTION OF FRICTIONAL HEAT IN
COMPRESSION

quantity 1–3, then a line through point 1 parallel to the final internal energy line will intersect line s–s on the vertical line for the temperature which must obtain, it being t_2.

Frictional heat also is generated during the expulsive part of the stroke, in quantity proportional to its length. If this added heat goes into the compressed gas, it increases its volume and raises its temperature. It is greater at the adiabatic than at the isothermal, as the expulsive portion of the stroke is longer at the former point than at the latter. Adding it to the heat of friction of the compression stroke, we find the sum to be constant. The total cylinder length is the same, regardless of the temperature of discharge, the constant being measured by the vertical distance between lines C–D and b–q. Then we lay

on the diagram the line $G-K$, which is at any temperature the same height above the normal work line $E-F$ that $s-s$ is above $C-D$, and then the line $g-q$ for resultant net total work which is measured in vertical line down to base $A-B$. For instance, at final temperature t_4 the resultant net total work is measured by distance 7–4, in B.t.u. of equivalent heat.

In the preceding paragraph it was assumed that the compressed gas parts with all heat of friction which is generated during its expulsion, it being a part of the total loss as measured by 3–13. However, the compressed gas may receive and retain the heat of friction incident to its expulsion. For this case, let the temperature as at the instant compression ceases be t_1, and let heat $d-g$ be generated by friction during expulsion and pass into the compressed gas, thereby enlarging the volume of gas which passes through the discharge valve and raising its temperature by a little, say to t_3. This heating of the gas does not in the least affect the work of expulsion as long as the pressure in the discharge main remains constant, and thus the resultant net total work still is measured by $g-e$, but t_1 is not the actual discharge temperature. A line through d parallel to the final *total* heat line through $O-H$, and a horizontal line through g intersect at a point m on the true final temperature vertical, the resultant work, measured from m to base $A-B$ being the same as $g-e$. Similarly, if at the instant compression ceases the temperature be t_4, the heat of friction during expulsion being 6–7, and this remaining in the gas, the discharge temperature is t_5 as fixed by point 8. It should be noted that in this instance the total of heat lost to jacket water, air, metal, etc., is that measured in B.t.u. by 2–3, not that measured by 13–3. Again, at this stage the gas might receive or lose more heat than that due to friction in expulsion. For instance, instead of heat quantity 6–7 it might receive 6–9, Sketch (B), in which case the temperature would rise to t_6; or more heat than 6–7 might be lost to jacket water, etc., the temperature falling to t_7 if the loss be 7–10. In both or any like cases, the resultant net total work is measured between the horizontal line $z-z$ and the initial heat base $A-B$. The impor-

tant temperature is the one obtaining as compression ceases, not the true temperature of discharge. The latter may be measured with fair accuracy, but the former cannot; and hence one may not know with exactness what the temperature gains or loses in the expulsion stroke. In ordinary operation it is perhaps safe to assume that gains offset losses, so that the discharge temperature is the same as that at the instant when compression ceases and the discharge valve opens.

Piston Friction in Expansion. — Taking now the case of expansion accompanied by piston friction, we refer to Figure 33, a straight-line heat-work diagram. Heat received from external sources during the expansive part of the stroke is measured vertically in the triangle $O-J-P$, for any final temperature found therein, as shown in Figure 24. The expanding gas does as much work as if there were no friction, but a part of that work is spent in overcoming the resistance due to friction, the work actually done on the moving piston being less accordingly. This frictional loss varies with the relative length of the expansive stroke, diminishing from the isothermal towards the adiabatic or lower temperatures. With a notation similar to that for the compression case, line $s-s$ is drawn to mark the frictional loss. At the adiabatic temperature the effective work done by the gas in the expansion part of the stroke is $L-5$, not $L-J$ as it would be were there no loss due to frictional resistance. Initial heat energy, first transformed into mechanical energy to overcome frictional resistance, reappears as heat of friction. This may be lost by radiation and conduction from the cylinder metal, or it may pass into the expanding gas. If it be lost and if this be the only heat loss, then the final temperature will be the same as the adiabatic, the expansion doing effective work $L-5$. If it remains in the gas and if aside from this there be no other gain or loss, then the consequent slightly greater length of travel increases the frictional heat from that measured by $J-5$ to that measured by $r-w$, the final temperature to t_1 and the effective work done to $c-w$. Work $c-w$ is greater than $L-5$, showing that, if the heat of friction be retained in the expanding gas, it is not wholly wasted, a portion being retrieved

in effective work done. Line K–G, at the same distance above F–E that s–s is above D–C, shows at any temperature the net total work as reduced by the portion spent against friction in the expansive part of the stroke. This is not the total cylinder friction, that of admission remaining to be considered.

Heat of friction developed in the course of the admission part of the stroke is a constant quantity, provided the initial temperature is constant. By initial temperature is meant that at
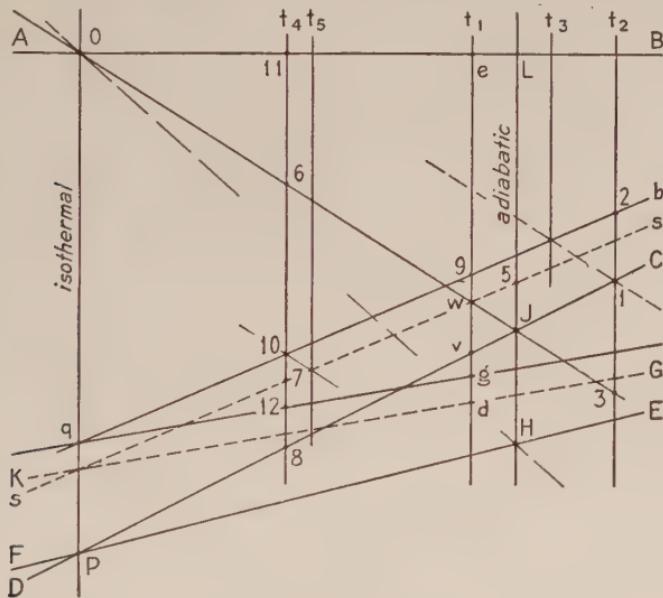
FIG. 33. DISTRIBUTION OF FRICTIONAL HEAT IN
EXPANSION

the instant the inlet valve is closed and expansion begins; we are concerned with this temperature, not that at the instant admission begins. To be sure, in an actual engine using gas, air, or steam expansively, the total length of cylinder and piston stroke are fixed, and to permit expansion to any given ratio with varying initial temperature, the point of cut-off must be variable; varying this point, however, varies also the weight of gas admitted, the final result per pound in work and friction being the same as in the assumed imaginary cylinder of varying length.

Returning to Figure 33, work equivalent to that of heat of the friction during admission is subtracted from that marked by line s–s, the subtractive term being the constant quantity 7–10, w–9, etc.   Line q–b then marks the resultant work of expansion, while the corresponding line q–g marks the resultant net total work done; this net total is that done in admission, plus that done in expansion, minus that required to force out the spent gas or exhaust of the preceding stroke and minus the work spent in overcoming frictional resistance through the complete stroke. At temperature t_1 this resultant work is e–g; at t_4 it is 11–12; etc.   At t_4 the heat received from sources other than friction is measured by 6–10, while 7–8 is gained from friction in the expansion stroke and 7–10 from friction during admission travel of piston.   This last we assume does not pass into the gas before cut-off, but passes instead into the cylinder and piston metal, lodges there a fraction of a second, and then passes into the expanding gas after closure of the inlet valve.   If it does not thus pass and if quantity 6–10 remains constant, then the final temperature is t_5 instead of t_4.   Any heat that enters the gas prior to cut-off and remains raises the initial temperature and thus establishes a different governing condition, requiring a different diagram.

The gas may lose heat by conduction while expanding, in such quantity that its final temperature shall fall below the adiabatic. For instance, if the total heat lost be 2–3, the discharge or exhaust temperature must be t_2.   If the surrounding air, metal parts, etc., can receive but a limited quantity of heat, such as 1–3, then a line drawn through point 1 parallel to the final *internal energy* line to intersection with line q–2 will intersect the discharge temperature which must obtain, that is, t_3.   Note the difference here as compared with the case of compression, page 105.   Here the final state of the gas is that after expansion, during which the temperature is modified by changes in internal energy; there the final state was that after passing the discharge valve, during which the gas was free to expand and any modification of temperature was due to change in total heat content.

General Graph as Modified by Piston Friction. — It is interesting to trace the effects of frictional resistance through the complete range of final temperatures, both practicable and speculative; and for this purpose a graph similar to Figure 29 is first drawn. (See Figure 34.) The work curves therein shown
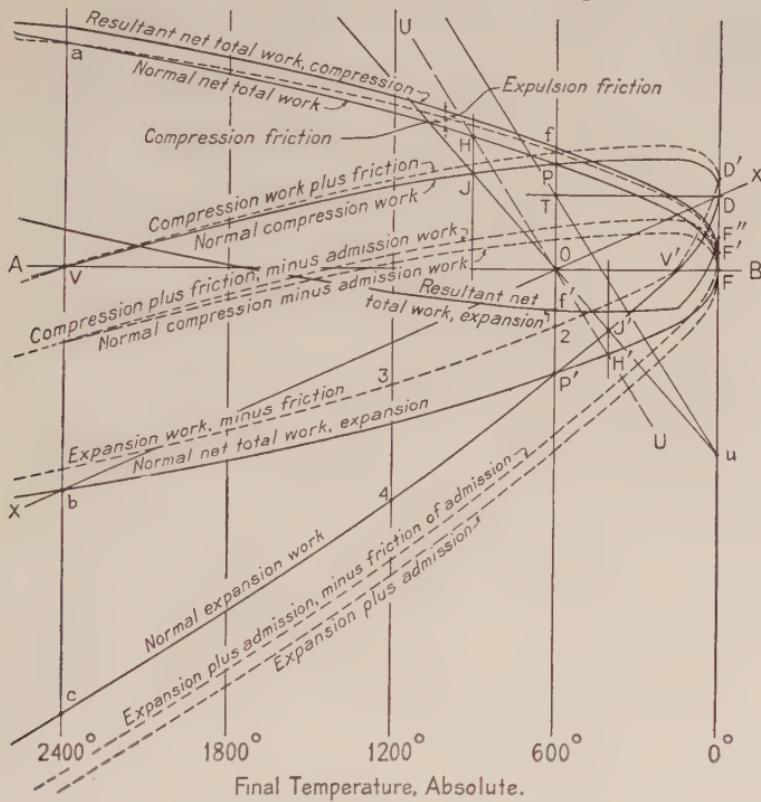
FIG. 34. THE GENERAL GRAPH FOR COMPRESSION AND EXPANSION, MODIFIED BY FRICTIONAL HEATING

have been plotted for air, pressure ratio 4, initial temperature 140° or 600° absolute, using the five-point method briefly outlined on page 97. Taking compression first, we find the extra work required for overcoming piston friction is a constant quantity, measured by $P-f$ at the isothermal, this being the sum of the frictional resistance for the compressive and expulsive parts of the stroke. Of these the first becomes equal to $P-f$ at

absolute zero temperature, at which point there is no expulsive work and hence no friction due thereto. It gradually decreases as the compressed volume decreases, until at the temperature for isovolumnic compression, 2400° absolute, it becomes 0.0, and there the friction of the expulsive part of the stroke becomes equal to $P-f$. To the left of this point the negative work is increased by friction, and this in turn diminishes the resultant net total work done by heat. (In reality the compression to higher pressure, in this speculative region, is accompanied by increase of volume; and the work is done wholly by the heat applied.)

Passing now to the expansion side, we assume for the moment that here the frictional heat is the same per unit of length of cylinder as in the compression region, although the mean effective pressure is lower and perhaps the friction should therefore be less. The length of cylinder (volume) occupied by a pound of air after admission to the expansion machine is exactly the same as the full length of the compressor cylinder, the initial pressure and temperature being the same in each; hence if friction per unit of length be the same, the friction during admission is equal to $P-f$ at the isothermal point. The normal admission work is diminished to this extent, as shown by the broken line curve parallel thereto and meeting the compression friction curve at F'. The friction for the expansive stroke is 0.0 at the isovolumnic point, V', and increases with higher temperatures. At the isothermal the air occupies four times its compressed or initial volume, the expansion travel being three times the initial volume, with friction $3 \times P-f$, as at $2-P'$. At 1200° absolute the friction of expansion travel is $7 \times P-f$, as at 3–4; at 1800° it is $11 \times P-f$, etc. To the right of the isovolumnic point the expansion work is negative and is increased by the work required to overcome frictional resistance; hence the net total work is not 0.0 at F, as in the normal case with no friction, but is negative work equivalent to that required to overcome friction in both the admission and the negative expansion strokes, the curve thus ending at F''. Combining the frictional losses for the two parts of the stroke, we get the re-

sultant curve shown. If we were to assume a smaller allowance per unit of length for friction in the expansion cylinder, the divergence between the normal and the modified curves would, of course, be less. In practice, the frictional resistance is not reduced in proportion to the mean effective pressure, and hence in general the work spent in overcoming friction in expansion machines is greater per pound of air or gas than in compressors.

CHAPTER VII

CYLINDER CLEARANCE SPACE AND ITS EFFECTS

Clearance in Compressors. — In preceding discussions it has been assumed that the cylinder of the machine was emptied of gas at each expulsion stroke, but in actual compression this is rarely done. Figure 35, Sketch (A), represents a typical compressor cylinder, although no attempt has been made to show
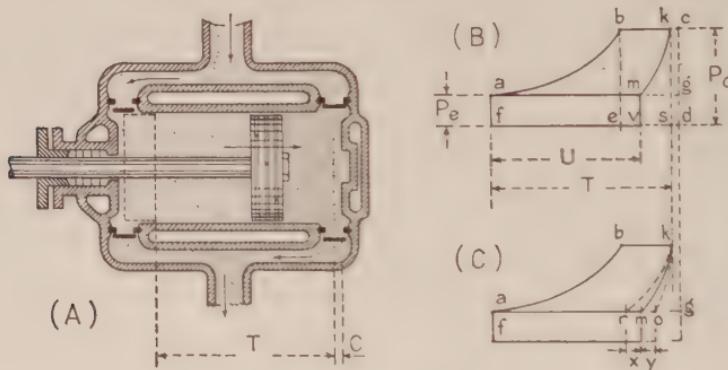
FIG. 35. CLEARANCE SPACE AND RE-EXPANSION IN COMPRESSOR

details of construction. The travel of the piston at each stroke is T. The outer face of the piston at the stroke end is at distance C from the cylinder head. A small quantity of compressed gas is in this space at the end of the stroke and remains there, re-expanding back to the initial low pressure when the piston recedes on the return stroke. Not only is space C filled with gas, but an additional quantity is entrapped in any valve pockets, nut recesses, or other cavities that may exist. Re-expanding, the gas helps to push the piston back to the position shown by m–v, Sketch (B), at which point its pressure has fallen to a little less than that in the manifold containing

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the low-pressure gas, and then the inlet valve opens.   Obviously, the length of cylinder that fills with low-pressure gas is that marked U , and not the length of stroke, T .   If the distance $s-v$ be one-eighth of T , the receiving capacity is reduced one-eighth by the presence of the clearance gas, the machine receiving at each stroke only seven-eighths of the total volume swept through by the piston.   The receiving capacity may be said to be seven-eighths, or 87.5 per cent, unless further reduced by other factors.   There are other factors, such as pressure-drop at the inlet valves, heating of the gas while passing through the inlet valves and while in the cylinder before the inlet valves close, slip of gas between piston rings and cylinder wall, and leakage at the several valves.   These factors are to be considered in the following chapter.   We shall now investigate the effect of clearance volume on the power required for compression.

Effect of Clearance on Work of Compression. — Taking a pound of any perfect gas — or air — let it be compressed, in any given ratio, the work being shown by the heat-work diagram of Figure 36, the compression work " curve " being the straight line $J-P$.   No expulsion work is done on any gas that may remain in the clearance space, hence for our present purpose we need not draw the curve for net total work.   Let a temperature for the compressed gas be selected at random, say t_2 , and let this remain constant during expulsion.   This, then, is the temperature of the gas found in the clearance space as at the instant that the piston starts on its return stroke.   During its re-expansion this gas does work, and its pressure falls until it becomes equal to or slightly less than that of the low-pressure gas on outer surface of the inlet valves.   At that point the temperature of the re-expanded gas may be equal to, less, or more than its temperature when re-expansion began.   If equal, the re-expansion has been isothermal, to temperature t_2 , and the work done per pound of gas re-expanded is measured by $P'-O'$, this also being equivalent to the quantity of heat which the gas has received from cylinder and piston metal during the re-expansion.   With final temperatures of the re-expanded gas

lower than t_2, the quantity of heat received from metal is less than at t_2, and is nil at the adiabatic point, while with temperatures below the latter point the gas loses heat.

During compression the gas yields to the cylinder metal or other substances the quantity of heat $a–O'$ and retains as added internal energy the quantity $e–O'$, the sum of the two being the heat equivalent of the work of compression. Now, if in the reexpansion of one pound of gas, it can be made to take back all
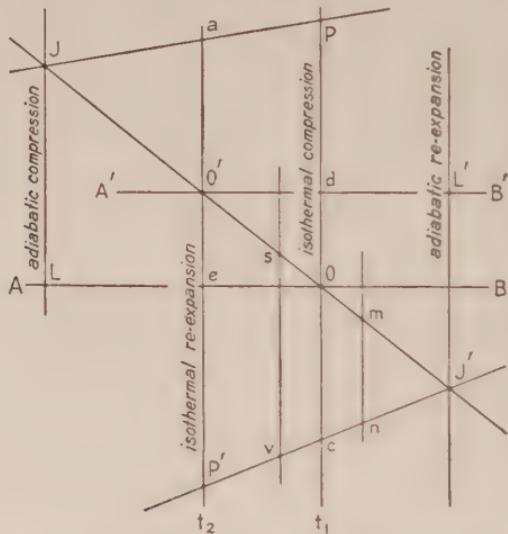
FIG. 36. COMPARISON OF WORK, COMPRESSION AND RE-EXPANSION

of the heat that was lost during compression to the cylinder metal, jacket water, etc., which is quantity $a–O'$, then the temperature must again become that at which compression began, t_1; and therefore $O–c$ must equal $a–O'$. Now $O–d$ is equal to $O'–e$; therefore $d–c = e–a$, meaning that the work done per pound of the re-expanding gas equals that done upon it during compression.

We may say, therefore, that aside from a little extra work needed to overcome the frictional resistance of the wasted part of the piston stroke, the existence of an ordinary clearance space does not add to the work of compression, provided that the heat

returned to the gas during its re-expansion be the equivalent of that it surrendered during compression. If some lesser quantity (per pound) be returned, such as $m-n$, then the temperature as after re-expansion must be lower than t_1 and the work done less than $a-e$. In this case, therefore, the presence of the clearance space adds to the work of compression, inasmuch as that work is not returned in full during re-expansion. On the other hand, if the heat gain per pound during re-expansion be greater than the loss during compression, such as quantity $s-v$ in the diagram, then the temperature as after the re-expansion must be higher than t_1 and the work done by re-expanding gas be in excess of that done upon it in compression. This is possible in an ordinary compressor, where the clearance space can contain a weight of gas which is but a small fraction of that compressed per stroke, this small quantity being in contact with a relatively large surface of hot metal. If, for instance, the clearance gas weigh one-tenth of the weight compressed per stroke, then only one-tenth of quantity $a-O'$ would need to pass to the cylinder metal and then from that back into the re-expanding gas in order to make the latter do work equal to that done upon it in compression; if more than that one-tenth should thus pass, it would reduce by a little the total work required for compression.

While we cannot know exactly what temperature fall occurs in the re-expansion, we may be quite certain that in the ordinary compressor it is to some point between the isothermal and the adiabatic. There is not, generally, any source of heat supply at a temperature above the former, nor any near-by substance at a temperature below the latter which might take from the expanding gas sufficient heat to bring its temperature to below or even to the adiabatic.

Again referring to the case of Figure 36, it is evident that the value of exponent n for re-expansion to temperature t_1 is the same as that for compression from t_1 to t_2. This being true, and the temperature change being equal in the two cases, formula (13), (14), or (18) would at once indicate that the work returned must equal that spent, per pound, as we have just proved by means of the heat-work diagram. Further, if

the re-expansion be to a temperature lower than t_1, then the value for exponent n is larger than that for the compression, and the formula would show the returned work to be less; and vice versa. Again, we may learn these facts from the diagrams of Figure 35, as follows:

Referring to Sketch (B), if the weight of gas in the cylinder be one pound and if the entire pound re-expand to the same pressure and temperature as before compression (beginning at b, no gas having been expelled) then the curve for re-expansion will be $b-a$, a retracement of $a-b$, with the same area under each, that area measuring the equivalent work for the strokes of piston. Now if less than the pound should re-expand, say the weight found in clearance space $k-c-d-s$, and again to the temperature as before compression, then the re-expansion curve will be $k-m$, and the work done will be in proportion to the weight of gas. Thus, if the weight in clearance space be one-tenth pound, then area $k-m-v-s$ will be one-tenth of area $a-b-e-f$; and it follows that the net total work required for compressing and expelling nine-tenths of a pound of gas is nine-tenths of what it would be for a full pound; and therefore, with the conditions as stated, the presence of the clearance space does not affect the power required per pound of gas delivered.

Next, suppose that the temperature of the re-expanded gas be lower than before its compression. Now the re-expansion curve takes the position $k-o$, and the area under this is less than one-tenth of that under $a-b$; hence the net work per pound of gas delivered is greater than in the former case or than if there were no clearance space. On the other hand, if the re-expanded gas be warmer than before compression, it will move the piston to point r, the area under the curve $k-r$ being greater than one-tenth of that under $a-b$; hence with this condition there is a saving in work per pound delivered. There is a retrieval of a small quantity of the work done in the compressive stroke, a utilization of energy which otherwise would have been lost as heat to the jacket water or by radiation.

It is probably fair to assume that, in general, no extra power is required on account of the presence of a clearance space, other

than the small amount needed to overcome the friction incident to the wasted part of the piston stroke. The relative unimportance of that friction may be shown thus: let the net total work per pound of gas compressed be 100 B.t.u., in heat equivalent, and let the frictional resistance for the machine increase this by 20 per cent, a liberal allowance, to 120 B.t.u. Now if, say, 10 per cent of the cylinder volume be taken by re-expanded gas, then the work done in re-expansion is 10 B.t.u., we will assume, and the net work is decreased to 120 − 10 = 110 *B.t.u.* for the 0.9 lb. of gas received into the cylinder and delivered. This is at the rate of 110/0.9 = 122.2 *B.t.u. per lb.*, only 2.2 more than the theoretical work as with no clearance.

In all of the cases above considered, the weight of incoming low-pressure gas is a constant quantity. The incoming gas mingles with the re-expanded gas and the temperatures of the two equalize to t_1 (as for Figure 36 and discussion). The volume of the incoming gas is that measured by U of Figure 35, and this regardless of whether the re-expansion curve ends at m, o, or r. If at o, the clearance gas takes enough heat from the incoming gas to cause its volume to increase from that measured by g–o to that measured by g–m, this change in volume taking place after re-expansion has ceased but prior to closure of the inlet valve. On the other hand, if the re-expansion be to point r, then heat is given by the clearance gas to the incoming gas and again the volume required by the former changes, shrinking from that measured by g–r to that measured by g–m. It is seen, then, that any shifting of position of the re-expansion curve, as to k–o or k–r, affects the work done in re-expansion but not the volume of gas received per stroke of the piston; provided always that the temperature attained by the mingled clearance and inflow gases be that first assumed, t_1, and that this be the temperature at the instant that compression begins.

Clearance Pockets in Compressors. — Builders of air and gas compressors endeavor to reduce clearance space to the least amount thought consistent with safe operation, so that there shall be no undue loss in cylinder capacity or extra work required to overcome frictional resistance. However, if the motive power

be a constant-speed motor, it may be advantageous to have adjustable "clearance pockets" in the compressor, the function of these chambers being to receive and hold a portion of the compressed gas, like a large clearance space, the re-expansion thereof causing a diminution of receiving capacity of the cylinder without change in initial or final pressure or speed in revolutions per minute. Our analyses have shown that, other conditions being equal, operation with such pockets should require but little more power, per pound of gas actually delivered, than operation without them.

Clearance in Steam or Air Engine Cylinder. — Figure 37, Sketch (A), showing the working cylinder and valve of a simple
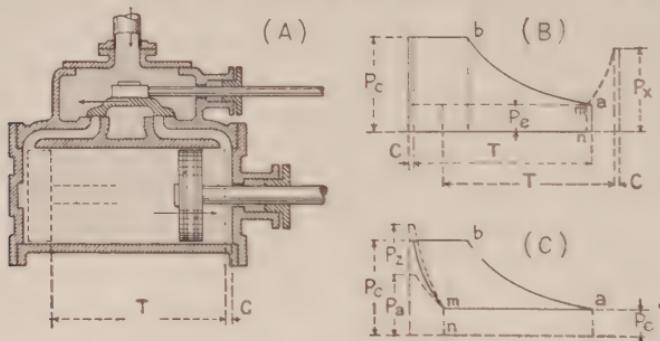
FIG. 37.   CUSHIONING SPACE AND RE-COMPRESSION IN CYLINDER OF STEAM OR AIR ENGINE

slide-valve engine, will serve in discussion of the effect of clearance in machines using a vapor, air, or gas expansively. In this, the exhaust valve was open until the piston reached the position shown, at which the moving slide-valve has just closed the exhaust port. The piston continues to the end of its travel distance, T, and re-compresses the entrapped steam into the clearance space C and the passage leading to the closed valve, the pressure of the steam gradually rising from that of the exhaust, P_e (Sketch B), to some indefinite higher pressure, P_x. The work done, relative to that of the expansion stroke, is again and more clearly shown in the combined diagram of Sketch (C). On the return stroke, the compressed steam re-expands, it having mingled with the high-pressure steam coming through the

inlet port and its share of the work done being a part of that measured under the curve $b–a$.

The steam which is compressed into the clearance space acts as a cushion to receive the impact due to the weight of the moving piston; the sudden stopping of which at the end of the stroke otherwise would cause excessive jar and wear on the cross-head pins and other parts. The valve adjustment may be such that the final cushion pressure, P_x, shall be equal to P_c, or it may be less, as P_a, or more, as P_z. If less, the incoming high-pressure
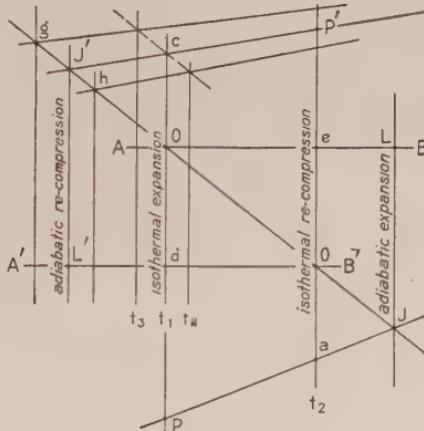
FIG. 38.   COMPARISON OF WORK, EXPANSION AND
RE-COMPRESSION

steam promptly compresses it up to its own pressure, with reduction of volume; if more, it raises by a little the pressure of the steam in the steam-chest, that pressure falling to the normal P_c shortly after the piston begins the succeeding admission stroke.

Effect of Clearance on Work of Expansion. — Figure 38 is a combination diagram for cushioning and expansion. We assume, first, that the setting of the valve is such that the cushion steam is compressed to pressure P_c and the same temperature as that of the steam in the steam-chest, which is t_1. Then, by reasoning analogous to that employed in the discussion of Figure 36, it can be proved that, whatever the temperature at exhaust, t_2, the energy spent in compressing the cushion steam

is equal to·that which the same weight of steam yielded in its earlier expansion against the piston, that is, that $c-d$ is equal to $e-a$ (one pound of steam in each instance). Now, if the temperature of the compressed steam be higher than first assumed, as for instance t_3, then its pressure also must be higher. We have a new and larger pressure ratio and a higher adiabatic temperature, that of point g, and clearly the work at temperature t_3 exceeds that measured by $c-d$. On the other hand, with a lower final temperature and consequent pressure the work would be less, as on temperature vertical t_4. In either case the temperatures of the cushion and incoming steam equalize during the admission stroke to t_2 at or before the point of cut-off, as otherwise we would have an initial condition of steam differing from that stated for our problem.

The steam which expands at each stroke of the piston is that admitted at the inlet port plus that which was found in the clearance space; and the greater this total, the longer the expansion stroke must be to allow the pressure to fall to that of the exhaust passages. Length of expansion stroke in the engine is varied by change in point of cut-off, which in turn affects the weight of steam admitted per stroke and the total piston travel per pound. Piston friction per pound of steam therefore increases with size of clearance space. This fact is of little importance, as the space must be proportioned to secure the necessary cushioning effect, regardless of friction. In actual engine operation, the valve setting is generally such that the cushion pressure is a little less than the steam-chest pressure, so that the presence of the cushion does not materially detract from the work done per pound of steam.

CHAPTER VIII

VOLUMETRIC EFFICIENCY OF COMPRESSORS

Reduction in Cylinder Capacity Due to Clearance. — It has been shown (page 113) that the presence of the gas re-expanded from a clearance space reduces the intake capacity of a compressor cylinder, the net capacity being constant with any given
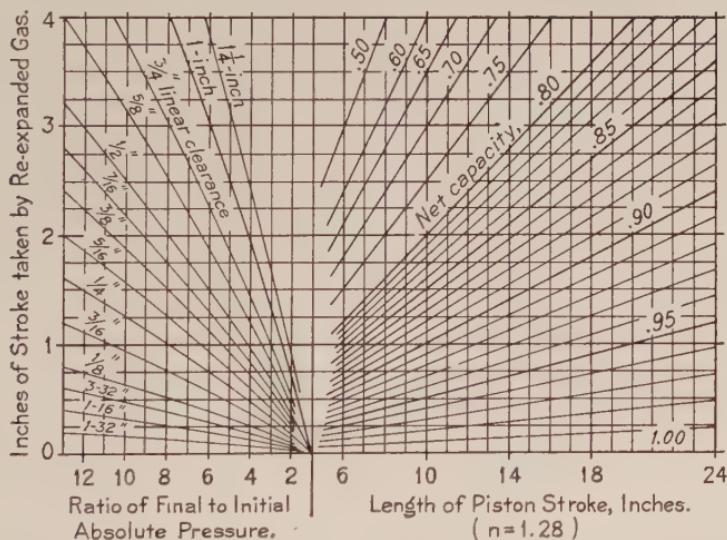

FIG. 39. CHART SHOWING NET CAPACITY OF COMPRESSOR CYLINDERS AFTER ALLOWANCE FOR CLEARANCE, IN PER CENT OF GROSS PISTON DISPLACEMENT

initial pressure and temperature. That is, referring again to Figure 35, with a given weight of clearance gas, volume U is constant, and the ratio of the net capacity to the volume swept through by the piston is U/T. It is to be borne in mind that the gas which occupies volume U is at the temperature and pressure at which compression begins, not at the temperature and pressure as found in the suction manifold.

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Figure 39 is a chart constructed to show the values of ratio U/T for compressor cylinders with length of piston stroke from 6 to 24 inches, inclusive, the value for exponent n being taken as 1.28. For a value of n less than 1.28, the curves in left-hand section of the chart would lie higher than as drawn, or lower for values greater than 1.28; in the former case, the capacity ratios would be less than as charted, in the latter case more. To use the chart, find the intersection of the pressure ratio vertical with the curve for linear clearance; then trace a horizontal line through this point to the right to intersection with the vertical line denoting the piston stroke. The diagonal through this last point gives the ratio U/T. For example, with pressure ratio 6 and linear clearance 5 16-inch, the ratio for a 10-inch stroke is barely 0.91; for a 14-inch stroke it is about 0.935; and similarly for any other case. If there are valve pockets or other recesses of considerable size, their estimated equivalent in linear clearance must be taken into account.

Reduction in Capacity Caused by Heating During Inflow. — Clearance space is not always the most important factor affecting the capacity of a compressor. Heating of the gas as it flows through the inlet valve and while in the cylinder prior to closure of the inlet valve may cause a greater loss in capacity. In the case of an ammonia compressor used in refrigeration work, the temperature of the discharged gas (vapor) may be something between 150° and 250°, and that of the engine room and its equipment 70° to 90°, while that of the ammonia vapor before the inlet valve of the compressor may be from 20° to as low as −30°. With such high discharge temperatures and warm machinery, the piston and the interior surfaces of the cylinder are necessarily quite hot; and the incoming cold vapor takes heat from these parts while the inlet valve is open, causing an expansion of its volume per pound. The temperature rise and consequent expansion are not so great in the air compressor, perhaps, as in that machine the air received is at a temperature not much lower than that of some parts, at least, of the compressor. In no case is it possible to learn the extent of this temperature rise by use of a thermometer; hence the best we

can do is to estimate it as closely as we may and compute the
effect on volume.   For example, suppose that an ammonia
compressor is working on vapor which comes to it, saturated,
dry, at 30 lbs. absolute pressure, at which its temperature is
− 0.6°, and let the discharge temperature be 170°.   With these
conditions, it is conservative to estimate that the temperature
of the incoming vapor after contact with the hot piston and
cylinder must be at least 30° when the inlet valve closes.   Now
the volume of the dry, saturated − 0.6° vapor in the suction
main is 9.236 cu. ft. per lb., according to the ammonia tables,
while that of the 30° vapor is 9.966 cu. ft. per lb.   It is plain
that the actual weight of vapor which will flow into a given
cylinder space, such as the net capacity U of Figure 35, is
only $\dfrac{9.236}{9.966}$ of the weight which that space might have received
had the vapor temperature remained − 0.6° as in the suction
main.   This weight ratio is 0.927.   If, then, the effect of a clear-
ance space has been reduction of the capacity of the cylinder to,
say, 96 per cent of what it would have been with no clearance,
and the inflow heating reduces that 96 per cent to 0.96×0.927
$= 0.8899$, then the volumetric efficiency of the machine after
accounting for these two of the several factors is 89 per cent.

**Reduction in Capacity Caused by Pressure-Drop at Inlet
Valve.** — There is another major factor — the drop in pressure
which occurs as the vapor or gas enters through the inlet valves
of the compressor.   This drop may be small if the valves are
light and properly adjusted, but can hardly be obviated al-
together.   Defects in valve design and adjustment or excessive
spring tension may make this factor abnormally large.   Con-
tinuing with the example last given, let us now assume that
while passing the inlet valve the vapor pressure falls from 30
lbs. absolute to 28 lbs., the temperature meanwhile rising to 30°
as before.   Now the volume per pound becomes 10.70 cu. ft. and
the net volumetric efficiency with the three factors accounted
for is

$$\frac{9.236}{10.70} \times 0.96 = 0.863, \text{ or 86.3 per cent.}$$

Cylinder Capacity Affected by General Condition Factor. —
After allowing as we have for the influences of clearance, inlet
heating, and pressure-drop, it is advisable to discount still
further the volumetric efficiency, due to what we may call the
general condition factor, in this way accounting for the effects
of slippage of gas past the piston rings and defects in valve
construction and operation.   The slippage may amount to but
little or may be quite serious with defective piston rings or
scored cylinder surface.   Any gas leaking past the piston not
only cuts down the effective receiving capacity but gives the
received gas an abnormally high initial temperature, which in
turn causes a higher discharge pressure and requires greater
power expenditure.   If the discharge valves do not close
promptly as the return stroke begins, a quantity of compressed
gas reënters the cylinder and has the same effect as if coming
out of a clearance space, except that re-expansion does not begin
until the valves actually close.   If the discharge valves do not
seat tightly, as in the event a bit of scale or gasket sticks to
disc or seat, then hot gas continues to reflow into the cylinder,
diminishing the quantity of cold vapor or gas that enters the
inlet.   In such a case the final temperature becomes higher with
each stroke, possibly to a point of danger to the compressor, and
the work is increasingly greater per pound of vapor or gas com-
pressed.   If the discharge valves do not open freely at the in-
stant after the pressure in the cylinder is equal to that in the
discharge manifold, the excess heat of compression passes to
the metal and from that to the incoming cold gas, causing
abnormal inflow heating and consequent increase in power
required.

Retarded opening of inlet valves causes an undue fall in
pressure for the first part of the suction stroke, with tendency
to remain too low throughout.   Retarded closing of inlet valves
allows expulsion of gas back into the inlet manifold, leaving a
lesser weight to be compressed, and, in some measure at least,
causing higher initial and final temperature and thus requiring
more work for compression.   To allow for all of these minor
factors, combined, it is convenient to make another percentage

reduction in volumetric efficiency. This may be as little as 3 or 4 per cent for a well-designed machine in first-class order, up to perhaps 15 per cent for a machine in bad, but workable condition. Using again our example in ammonia compression, we may fairly allow 7 per cent discount for general condition, the condition factor thus being 0.93; and now, with all factors taken into account, the true volumetric efficiency of the compressor is

$$0.93 \times 0.863 = 0.803, \text{ or } 80.3 \text{ per cent.}$$

Volumetric Efficiency, Ammonia Compressors. — Table V lists estimated volumetric efficiencies of ammonia compressors, for various working pressures (gage pressures) for well-designed machines in good order. It is to be understood that these efficiencies may be exceeded with some machines and favoring conditions.

TABLE V

VOLUMETRIC EFFICIENCIES OF AMMONIA COMPRESSORS

AT SELECTED WORKING PRESSURES

Back Pressure, Gage	Condensing Pressures, by Gage, Lbs. per Sq. In.						
	125	140	155	170	185	200	215
30	.865	.860	.855	.851	.847	.843	.839
25	.852	.847	.842	.838	.834	.830	.826
19.6	.838	.833	.828	.823	.819	.815	.811
15.7	.823	.820	.816	.811	.807	.802	.797
10	.798	.792	.787	.782	.778	.774	.771
5	.758	.752	.746	.741	.736	.731	.727
0	.716	.710	.704	.699	.694	.689	.684
4″ vac.	.689	.683	.676	.670	.665	.660	.655

Volumetric Efficiency, Air Compressors. — Air or gas compressors that receive at normal atmospheric temperatures, say 50 to 90 degrees, may be expected to show volumetric efficiencies somewhat greater than the ammonia compressor, provided discharge temperatures are maintained about the same. Thus

with moderate pressure ratios and with water-cooled cylinder, the efficiency may be from perhaps 80 per cent to as high as 94 per cent, depending on the influence of the four factors discussed.

Influence of Efficiency Factors on Work of Compression. — Clearance, as discussed on page 114, affects the power requirement but little. We may say that an allowance of 2 to 3 per cent additional is ample for the average case. Initial heating of the low-pressure gas during inflow calls for greater
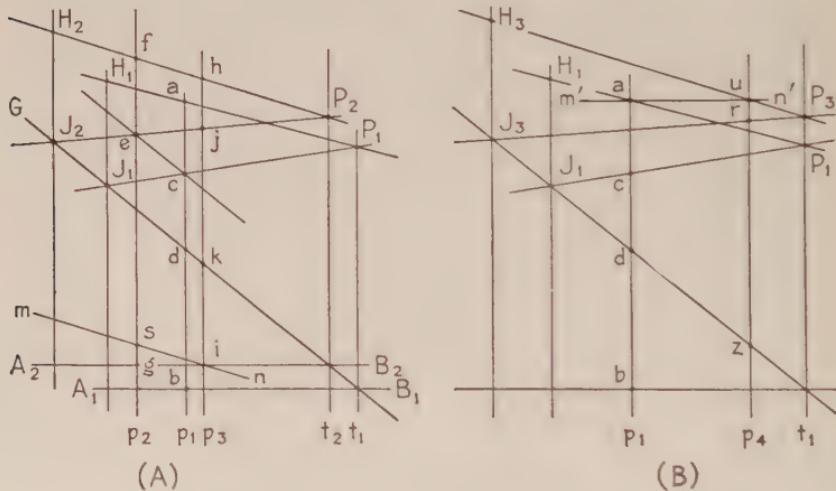

FIG. 40.   INFLUENCE OF INFLOW HEATING ON WORK
OF COMPRESSION

power expenditure, unless compensated by an increased flow of heat to the jacket water, as shown in Figure 40, Sketch (A). This embodies two heat-work diagrams, the first for an initial temperature of t_1, the initial heat line being $A_1–B_1$, the jacket and heat radiation efficiency being such that discharge is at temperature p_1, the net total work required, friction neglected, being measured in B.t.u. at $a–b$. Now, second, let the initial temperature rise to t_2, requiring the higher initial heat base line $A_2–B_2$ and higher net total work line $H_2–P_2$. Now if the heat lost to water, air, etc., remain as in first case, measured by $c–d$, then the line $c–c$ drawn through c parallel to $G–d$ cuts $J_2–P_2$ at

c at the discharge temperature that must obtain, p_2, at which the net total work is f–g, somewhat greater than a–b. If we measure from point f on f–g a heat quantity equal to a–b, thus locating point s, and through this point draw m–n parallel to H_2–P_2, it will cut base line A_2–B_2 at i at the temperature that must be attained if the net total work of compression is to be equal to that for the first case, it being temperature p_3, and at this the loss of heat to jacket water, etc., is j–k. Obviously, if c–d represents the best that the water jacket can do, the required work is increased with the increase in initial temperature, and in actual compressor operation this is the effect that may be expected to be caused by heating of the gas or air during inflow to the cylinder.

Figure 30 and the discussion thereof showed how the net total work curve rises with increasing pressure ratios. With a constant final pressure, a drop in pressure at the inlet valve means an increase in ratio of pressures, in general with increase in the work of compression. The pressure drop accompanies inflow heating, and their combined effect is to raise the net total work curve still higher than as shown in Sketch (A), throwing the temperature vertical for p_2 to the left and p_3 to the right of their marked positions. If we consider pressure drop alone, Sketch (B) shows that in order to maintain the work measurement the same as at a–b the final temperature must be brought down to p_4, determined by the point u where a horizontal line m'–n' drawn through point a cuts the curve H_3–P_3, requiring heat loss r–z, much in excess of c–d.

Whatever the pressure drop and the initial heating may be, if we can measure or estimate them with accuracy and thus obtain the actual initial conditions as to pressure and temperature and can learn the actual final conditions as well, a heat-work diagram drawn for the data thus found will show truly the work required. Any valve defects or slippage past the piston affects pressures or temperatures, or both; and in clearly defined cases it may thus be found possible to take these items into account in construction of the diagram. In general, however, it is better and far simpler to allow for the clearance and general

condition factors in one percentage addition to the required work as without them, and this before making any addition to cover extra work required to overcome frictional resistance of piston rings, bearings, pins, etc. This percentage allowance for general condition may be from 4 to 10, depending on condition of the machine.

CHAPTER IX

HEAT–WORK AND INDICATOR DIAGRAMS COMPARED

Indicator Diagram. — The figure traced by the pencil of an ordinary engine or compressor indicator on an indicator "card" is in reality a pressure-volume diagram. The piston of the instrument is subjected to the varying pressure which obtains in the cylinder throughout one or more complete cycles,
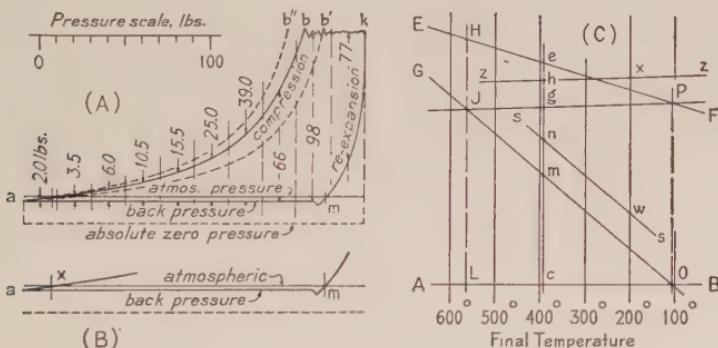

FIG. 41.   COMPARISON OF INDICATOR DIAGRAM WITH
THE HEAT-WORK DIAGRAM

the pencil tracing the compression curve, expulsion line, re-expansion curve, and the back-pressure or contra-work line, for the compressor, or the admission line, expansion curve, exhaust pressure line, and cushioning curve for the steam or air engine. The piston is then relieved of the cylinder pressure and the pencil allowed to trace a straight line which marks the atmospheric pressure. A spring of known tension is used in the instrument, so that vertical distances on the card measure pressures at a definite number of pounds per square inch, while the horizontal length of the diagram corresponds to length of piston travel. An indicator diagram such as might be

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traced by the instrument on an air compressor is shown as Sketch (A) of Figure 41. It may be noted that the back-pressure line is barely below the atmospheric, showing that there is but little drop in initial pressure at the inlet valve. The diagram gives no information as to the heating of the air while passing the valve or after admission to the cylinder.

In order to use the indicator diagram to find the work done, the length of piston stroke, diameter of cylinder bore, and number of revolutions per minute must be known, also the spring rating. The area enclosed by the several curves and lines, $a–b–k–m$, represents the work done during one active stroke of the piston. Vertical distances between the curves represent pressure in pounds per square inch and the length of card represents piston stroke in feet; hence the card area represents work done in foot-pounds for one piston stroke. Then with the work for one stroke known, multiplication by the number of strokes per minute and division of the product by 33,000 gives the horse-power actually expended on the air. The well-known formula is

$$I.H.P. = \frac{P \times L \times A \times N}{33,000}.$$

In this formula

$I.H.P. = Indicated\ horse\text{-}power,$

$P = Mean\ effective\ pressure\ against\ the\ piston,\ in\ pounds\ per\ square\ inch\ of\ its\ net\ area,$

$L = Length\ of\ piston\ stroke,\ in\ feet,$

$A = Net\ area\ of\ piston,\ in\ square\ inches,\ and$

$N = Number\ of\ strokes\ of\ piston\ per\ minute.$

The product of $P \times L \times A$ is the work in foot-pounds for one stroke. If the compressor be a single-acting, one cylinder machine, N is the same as the number of revolutions per minute. If a double-acting machine, the card gives the work for the end of the cylinder at which it was taken. Cards taken from the two ends of a double-acting cylinder should show substantially the same mean effective pressure; but area A of

piston face differs by the area of the piston rod, which must be deducted for the crank end. Then if the average area be used in the formula, the number of strokes per minute is, for the double-acting cylinder, $2 \times r.p.m.$

Let us say that the card of Figure 41, (A), is from one of two single-acting cylinders of a compressor, 7-inch bore, 12-inch stroke, and running at 116 revolutions per minute; and that the spring scale is the one shown, for convenience, above the diagram. The common method for finding mean effective pressure from a card is to divide its length into ten equal parts, then scale the vertical distance between curves or lines at the center of each space, as in this card where the heights scale 2.0 lbs., 3.5 lbs., 6.0 lbs., etc. Then the sum of these ten measurements divided by ten gives the approximately correct mean net pressure for the stroke, being the "mean effective pressure" required for use in the horse-power formula. Of course, the greater the number of divisions the more accurate the result obtained is likely to be; division into ten parts is merely for convenience. We get 34.3 lbs. as the mean effective pressure (m.e.p), and substituting this and the other numerical values in the formula, we have:

$$\frac{34.3 \times 1 \times 38.48 \times 232}{33,000} = I.H.P. = 9.28.$$

This indicated horse-power, it is to be understood, is the power actually spent in the compression and expulsion of the air. It does not include any power required to overcome frictional resistance in any part of the machine.

As has been stated, the data needed for finding horse-power by use of the indicator instrument are the indicator cards themselves, carefully taken, using a spring of known rating, knowledge of the piston diameter and stroke, and observation of the number of revolutions per minute. Given these data, a careful scaling and computation of the card area and use of the formula give the indicated horse-power. Temperatures, pressures, volumetric efficiency, and weight of air compressed per stroke or per minute do not enter directly into the problem,

but the last two items must be known before one can find the horse-power required per unit of weight or volume of air. The card itself is the record of pressures. Contrasted with these requirements for indicator work, construction of a heat-work diagram as described in these pages requires observation or careful estimate for the initial and the final temperatures and pressures, that the weight of air passed per minute through the discharge valves be known, also the general condition of the machine. The final temperature may be learned by use of a thermometer in most cases, not precisely, but with a degree of accuracy sufficient for the purpose. Gages show the initial and final pressures, to which the atmospheric pressure must be added to give the corresponding absolute pressures. The initial temperature, unfortunately, is difficult to learn and usually must be estimated, adding to the observed suction main temperature whatever inflow rise is thought proper. The weight of gas or air handled per minute must be measured or estimated, taking all modifying factors into account. Assuming that we must estimate the weight of air compressed per minute, the heat-work diagram for the preceding case will next be discussed.

Let the atmospheric pressure be 14.5 lbs., absolute, and the temperature 75°, while the discharge gage shows 96 lbs. and a thermometer in discharge line shows that the final temperature is 390°. Let us say that the valves are light, free-acting, and in good condition, that the piston is in excellent order and the lubrication efficient and satisfactory. With these conditions we may estimate that the actual initial pressure in the cylinder is 14 lbs., and that of the compressed air 112 lbs., absolute, thus making the ratio of compression exactly 8. With discharge at 390° the piston and cylinder are likewise quite warm and impart heat to the incoming air, so that the temperature of the latter at the instant compression begins can hardly be less than 100° or more. Let us estimate it as 105°. We now make use of Figures 18 and 19 to find the adiabatic discharge temperature with the work at that point and at the isothermal point, then draw the heat-work diagram, (C), of Figure 41.

Scaling the net total work as at 390° discharge temperature, we find it to be 98.5 B.t.u., in heat equivalent, per pound of air. To compensate for the small error involved in use of straight line E–F instead of the true curve we may add 0.5 and call the work 99 B.t.u. It is well to check this by use of formula (18), which gives 99 as the correct value. It remains to estimate the weight of air compressed per minute.

It has been stated (page 126) that the volumetric efficiency of an air-compressor cylinder may be 80 to 94 per cent of the volume swept through by the piston. In this case let it be estimated as 86 per cent. Then as the actual volume for 7-inch diameter and 12-inch stroke is 461.8 cubic inches, or 0.267 cubic feet, the volume of free air which enters at each stroke is $0.267 \times .86 = .2236$ *cu. ft.* Now, using the basic formula of page 12, $V = RT/P$, we find that a pound of the free air occupies 13.66 cubic feet. The number of piston strokes required to handle this volume is $13.66/.2236 = 61.09$; hence the number of pounds compressed per minute by both cylinders, at 116 r.p.m., is $232/61.09 = 3.797$. Then, as one horsepower is the equivalent of 42.45 B.t.u. of heat energy, we have

$$H.P. = \frac{99 \times 3.797}{42.45} = 8.855.$$

It remains to add an allowance for condition, for which 4 per cent or 5 per cent should suffice in this case. Adding 4 per cent, $8.855 \times 1.04 = 9.21$ *H.P.* Adding 5 per cent, $8.855 \times 1.05 = 9.30$ *H.P.*, the latter agreeing substantially with the horsepower as computed from the indicator card.

The indicator card is a graphic record of work actually done on the air by the piston at one stroke in a particular cylinder. The weight of air compressed per minute must be found by some other means. The heat-work diagram shows the work required per pound of air, independent of dimensions of cylinder and time required for compression. If both the indicator card and heat-work diagram could be made and used with absolute accuracy, they would yield results precisely equal, in any given case.

The location of point m, Sketches (A) and (B) of Figure 41, gives a rough indication of the volumetric efficiency. The re-expansion curve crosses the atmospheric line at about .88 by the volume scale. However, the back-pressure is a little below the atmospheric; and therefore the inlet valve cannot close until the piston has moved a short distance on its compression stroke, as to point x, Sketch (B), at which point the pressure inside has become at least as high as that outside in the inlet pipe. The indicator card may not reveal this condition, partly because the width of the marks made by its pencil is such as to obscure point x and partly because the " throw " of the indicator arm at this point may make the record somewhat inaccurate. The net volume as measured between points x and m might be the true volume received per stroke, but it would be the volume as after heating during inflow, hence not the true volume of free outside air which must measure volumetric efficiency. Moreover, a part of the heat imparted during inflow comes from the re-expanded air from the clearance space, meaning that that air is cooled a little and the net effect is the same as if the point m were moved slightly to the right. Knowing these facts and making due allowance for their effects, we may, in some cases, determine the volumetric efficiency from an indicator card with fair accuracy.

Adiabatic or Isothermal Compression or Expansion Curve. — For purposes of comparison it is sometimes desired to add the adiabatic or the isothermal curve, or both, to the graph made by an indicator pencil. Table VI will facilitate this. The initial absolute pressure is to be multiplied by a series of factors read from the table, if the case is compression, or divided thereby if for expansion. For example, if the initial pressure before compression be 14 lbs. and the isothermal compression curve is to be plotted, multiply 14 successively by the factors for volume percentages 90, 80, 70, etc., from the column headed $n = 1.00$. Thus at 30 per cent, meaning the stage where the compressed volume is only 30 per cent of the initial volume, we have $14 \times 3.333 = 46.7$ *lbs.*, absolute, per square inch. Through the several points thus determined, we

TABLE VI

FACTORS FOR INTERMEDIATE PRESSURES FOR USE IN PLOTTING COMPRESSION OR EXPANSION CURVES ON INDICATOR DIAGRAMS

Per cent of Vol.	Values for y or n								
	1.00	1.10	1.20	1.25	1.30	1.35	1.40	1.50	1.60
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
90	1.111	1.123	1.134	1.140	1.147	1.153	1.159	1.171	1.196
80	1.250	1.278	1.307	1.321	1.337	1.352	1.367	1.397	1.429
70	1.429	1.418	1.535	1.563	1.591	1.615	1.648	1.708	1.770
60	1.667	1.754	1.846	1.894	1.943	1.994	2.045	2.152	2.265
50	2.000	2.143	2.297	2.379	2.462	2.549	2.638	2.828	3.031
45	2.222	2.407	2.607	2.713	2.823	2.939	3.059	3.313	3.588
40	2.500	2.739	3.002	3.144	3.291	3.445	3.607	3.953	4.331
35	2.857	3.174	3.525	3.715	3.914	4.126	4.348	4.829	5.363
30	3.333	3.759	4.240	4.503	4.782	5.079	5.394	6.084	6.863
28	3.571	4.056	4.607	4.909	5.231	5.576	5.941	6.748	7.672
26	3.846	4.400	5.035	5.387	5.761	6.163	6.592	7.543	8.630
24	4.167	4.806	5.544	5.954	6.393	6.866	7.374	8.506	9.811
22	4.545	5.289	6.153	6.637	7.160	7.722	8.330	9.692	11.270
20	5.000	5.873	6.898	7.477	8.104	8.783	9.518	11.183	13.133
18	5.556	6.595	7.829	8.530	9.294	10.127	11.034	13.096	16.204
16	6.250	7.507	9.017	9.882	10.829	11.867	13.008	15.621	18.767
14	7.143	8.696	10.583	11.676	12.881	14.215	15.682	19.092	23.238
12	8.333	10.301	12.733	14.159	15.740	17.502	19.459	24.054	.....
11	9.090	11.336	14.135	15.782	17.625	19.681	21.976	.....	.....
10	10.000	12.589	15.819	17.783	19.952	22.388	.....	.....	.....
9	11.111	14.135	17.981	20.284	22.878	.....	.....	.....	.....
8	12.500	16.092	20.715	23.504	.....	.....	.....	.....	.....

draw the isothermal curve, $a–b'$, Figure 41. A similar series of multiplications by factors taken from the column headed 1.40 determines the points through which the adiabatic curve $a–b''$ is drawn. If we were to compute the horse-power as to these curves, we should get results agreeing closely with the work as measured at $O–P$ and $L–H$ of the heat-work diagram and increased by the small condition factor as before.

The heat-work diagram, unlike the indicator diagram, gives information as to the quantity of heat which is lost to jacket water, metal, and the air of the engine room. Here we must take into account the heat of friction in the cylinder. On the

diagram of Figure 41 the line z–z marks a 12 per cent addition for friction, being about 12 B.t.u. In estimating friction it is better to err by using too great an allowance, rather than too little. Now we assumed the inflow heating to be through 30°, which requires $30 \times .2396 = 7.2$ *B.t.u.* of heat per pound of air. The heat lost from the pound of air during compression to 390° is g–m, about 30 B.t.u. (by scale), and of this we may say that 14 per cent, corresponding to the .14 lb. of air in the clearance space, passes to the latter while it re-expands (page 115). Further, an indefinite quantity of heat is radiated from the warm cylinder to the air of the room and conducted to the machine frame, foundations, etc.; let us allow 4 B.t.u. for this item, per pound compressed. Then the sum of $7.2 + 30 \times .14 + 4.0 = 15.4$ *B.t.u.* is heat loss which is not to jacket water, and this subtracted from the total heat loss, h–m, which is about 43 B.t.u., gives as the remainder the heat which does pass to jacket water, h–n, which is 27.6 B.t.u. The sum of the other losses, 15.4 B.t.u., being marked as constant for all discharge temperatures by the line s–s, parallel to G–O, it is evident that if the discharge temperature is to be brought down to, say, 200°, heat as measured by x–w must be imparted to the jacket water. The conductivity of heat through the cylinder metal varies as the difference between the temperature of jacket water and that of the air in the cylinder; hence if the latter is to be brought down to approach the isothermal, the water supply must be colder, or more water must be circulated per minute, or more of the heated cylinder metal must be exposed to the water. To obtain the best results not only the barrel of the cylinder, but some portion of the heads as well must be jacketed, with all chambers and piping of generous size.

To complete the problem in horse-power computation, we should find the power required per 1000 cubic feet of free air compressed. Turning to Figure 20, we find that at the atmospheric pressure and temperature assumed, 14.5 lbs. and 75°, the weight of 1000 cubic feet of air is 73.3 lbs. We found that, not including friction, the compression of 3.797 lbs. per minute

required 9.30 *H.P.* If we take 16 per cent as a fair allowance for total machine friction, then

$$\frac{9.3 \times 1.16 \times 73.3}{3.797} = 208.3 \; H.P. \; per \; 1000 \; cubic \; feet \; of \; free \; air.$$

The quantity of heat lost to jacket water, etc., may be computed easily without the aid of a heat-work diagram. Thus in the example used in preceding paragraphs, the gain in internal energy when air is heated, by any means whatsoever, from 105° to 390° is $(390 - 105) \times .1711 = 48.8$ *B.t.u. per lb.*, this corresponding to the dimension m–c of our heat-work diagram. Then subtracting this from the computed work of compression, which is found to be 79.5 B.t.u., corresponding to g–c of the diagram, and adding the 12 B.t.u. for friction in cylinder, we get 42.7 B.t.u. as the total of the heat losses, the distribution being the same as before. It should be stated here, perhaps, that to compute the work of compression we use Figure 28 to find the proper value for n, then compute the work by a formula which is No. (18), except that the multiplication by n is omitted.

Work per Ton of Refrigeration. — The equivalent of a standard " ton " of refrigeration is the removal of 288,000 B.t.u. of heat in 24 hours, which is at the rate of 12,000 B.t.u. per hour or 200 B.t.u. per minute. The heat of fusion of ice is 144 B.t.u. per pound, therefore a ton (2,000 lbs.) of ice requires 144 $\times$ 2,000 = 288,000 *B.t.u.* in melting. The commercial term " ton of refrigeration " thus originates. A pound of the liquid refrigerant, entering the vaporizing coils at a given temperature and emerging as dry, saturated vapor, will take up in vaporizing a definite number of B.t.u., this being its refrigerating capacity per pound. For example, the heat of liquid of ammonia at 86°, corresponding to 169.2 lbs. absolute pressure as at the saturation point, is 138.9 B.t.u. per lb., according to the Bureau of Standards Tables. If this ammonia liquid be vaporized at 5° temperature, corresponding to 34.3 lbs. absolute pressure, the total heat of the vapor then is 613.3 B.t.u., it having taken up $613.3 - 138.9 = 474.4$ *B.t.u.* of heat in the coils where its work was done. Then in order to do refrigerating

work at the rate of one ton per day, the weight of ammonia vaporized thus per minute in the coils, if to dry, saturated condition, need be only

$$200/474.4 = 0.4216 \ lbs.$$

If the vapor leaving the refrigerating coils be wet, say of 94 per cent quality, then its total heat is less than 613.3 B.t.u. per lb., being less by a quantity equal to 6 per cent of the heat of vaporization; that is

$$total \ heat = 613.3 - (565 \times .06) = 579.4 \ B.t.u. \ per \ lb.,$$

and hence in this instance the refrigerating capacity of the one pound of ammonia is $579.4 - 138.9 = 440.5 \ B.t.u.$, and the weight which must be circulated per ton of refrigeration is

$$200/440.5 = 0.4542 \ lbs.$$

Table VII shows the weight of dry, saturated ammonia vapor which must be used per ton of refrigeration, for a few selected condensing and vaporizing pressures. (Gage pressures.)

TABLE VII

POUNDS OF DRY, SATURATED AMMONIA VAPOR REQUIRED PER MINUTE PER TON OF REFRIGERATION

AT SELECTED CONDENSING AND VAPORIZING PRESSURES. (GAGE)

Vaporizing Pressure, Gage	Condensing Pressures, Gage, Pounds per Square Inch							
	130	140	150	154.5	170	180	185	200
30	.4094	.4133	.4169	.4184	.4240	.4275	.4292	.4341
25	.4106	.4144	.4181	.4198	.4253	.4288	.4306	.4356
19.6	.4123	.4162	.4199	.4216	.4272	.4306	.4325	.4374
15.7	.4136	.4175	.4212	.4230	.4285	.4321	.4339	.4389
10	.4160	.4199	.4237	.4254	.4311	.4346	.4365	.4416
5	.4186	.4225	.4264	.4282	.4399	.4375	.4394	.4446
0	.4220	.4260	.4299	.4317	.4376	.4413	.4432	.4485
5″ vac.	.4243	.4284	.4323	.4340	.4399	.4437	.4457	.4510
9″ vac.	.4262	.4303	.4343	.4361	.4421	.4459	.4479	.4532

The following example with the heat-work diagram shows the ease with which the power per ton of refrigeration may be found, also the importance of taking inflow heating and pressure-drop into account. Ammonia is assumed to come from vaporizing coils at 5° temperature, saturated, which means at 34.3 lbs., absolute, or 19.6 lbs., gage pressure; and then to be compressed to 154.5 lbs., gage pressure, 169.2 lbs., absolute. First a diagram is constructed with no allowance for heating at valve or for pressure-drop, assuming that compression actu-
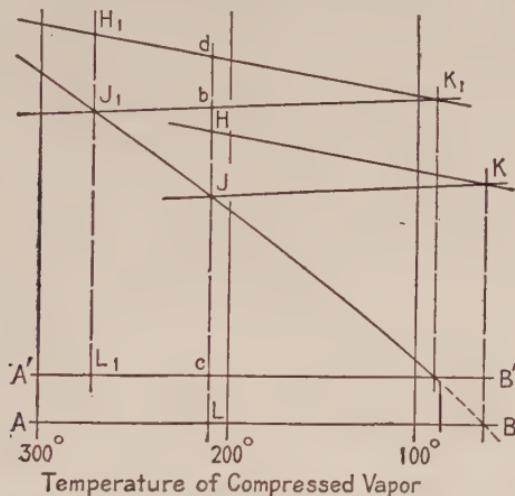
FIG. 42.   INFLUENCE OF INFLOW HEATING AND PRESSURE
DROP IN COMPRESSION OF AMMONIA VAPOR

ally begins on the vapor at 5° temperature and the full 34.3 lbs. pressure. See Figure 42. The adiabatic compression temperature is 209.8°, at which the work of compression is found to be 77.5 B.t.u. per lb. (measured as L–J in the diagram) while the net total work, L–H, is 99.6 B.t.u. per lb. (Properly speaking, these are the heat equivalents of the work.) Then using Table VII, the H.P. per ton of refrigeration is

$$\frac{99.6 \times .4216}{42.45} = 0.989.$$

Now we shall add the lines of a second heat-work diagram, assuming this time that the initial temperature is in fact 40°

and the initial pressure only 32 lbs., and that the discharge
valve does not open until the pressure in cylinder has risen to
172 lbs. The line for initial internal energy takes the higher
position $A'-B'$. The curve for final internal energy so nearly
coincides with that for the other case that but one line appears
in the diagram. The adiabatic temperature for this case is
about 272°, at which the net total work per lb., L_1-H_1, is the
equivalent of 116 B.t.u. If the actual discharge temperature
should be 209.8°, the same as the adiabatic for the other case,
then the net total work would be measured by $d-c$, 108.4 B.t.u.,
and the work per ton of refrigeration

$$\frac{108.4 \times 0.4216}{42.45} = 1.089 \ H.P.$$

Again, if the actual final temperature should be, let us say,
240°, as it might be if but little or no water were circulated
through the water jackets, then the power required per ton
would be 1.117 H.P. If to that we should add for a 5 per cent
general condition factor, the result would be 1.117×1.05
$= 1.173 \ H.P.$, which should agree closely with the I.H.P. as
computed from an indicator card taken for a cylinder working
under the conditions assumed. The actual power consumption
must include that for overcoming all machine friction resist-
ance, perhaps 16 per cent additional for the case in hand. Then

$1.173 \times 1.16 = 1.36 \ H.P. \ per \ ton \ of \ refrigeration.$

CHAPTER X

WET COMPRESSION

Purpose and Methods of Wet Compression. — The cylinder of a compressor is designed primarily for compression, any possible cooling of the gas or air contained therein being a secondary matter. Cooling by means of water jackets and by radiation is of value; but at best the discharge temperatures are fairly high, with ordinary working pressures, requiring greater power expenditure than with low discharge temperatures and complicating the problem of lubrication. One method of attaining the desired low final temperature is by working the compressor " wet," injecting a small quantity of liquid into the cylinder with every stroke of the piston. The liquid may be the condensate of the vapor being compressed, as in the case of wet compression of ammonia in a refrigerating system, or cold oil may be used, or cold water as in certain types of air compressor. In all cases the function of the liquid is to absorb a portion of the heat of compression and thus modify the final temperature. In the case with ammonia or other refrigerant, the liquid takes heat as latent for its own vaporization; oil and water take heat as sensible heat, the temperature being raised thereby; or they may vaporize if conditions favor.

In a refrigerating system a little more liquid may be fed to the room or tank coils than can fully vaporize therein, so that the vapor reaches the inlet valve of the compressor carrying in suspension more or less liquid in the shape of fine, floating particles. The use of such vapor, by intention, is also wet compression, in the broad sense of that term; but it is done in an ordinary compressor, whereas wet compression proper is done in a machine with special arrangement for the purpose. The machine designed for wet compression must be operated

at moderately slow speed with usually a relatively long piston stroke to give the liquid time for vaporization or heating.

Wet Compression of Air. — This is seldom done in the United States but is, or has been, common practice in Europe. The heat-work diagram is normal, similar to Figure 14, provided that none of the injected water vaporizes in the cylinder. The water is the substance to which heat is imparted, since it is external in the sense that it is not the material being compressed. If the water vaporizes to any extent, the case becomes somewhat like that of ammonia compression with liquid injection, as treated in a later paragraph, but is further complicated by the fact that we have to deal with the heats of two substances instead of one. If used in sufficient quantity, the water will take enough heat to hold the air temperature fairly low and thus avoid any considerable vaporization of water in the cylinder. The presence of moisture in the compressed air often is objectionable, and in such cases it must be trapped or baffled out before reaching the air engine or tool in which it is used, especially in those cases where the temperature of the expanded air falls below the freezing point of water.

Compression of Wet Vapor. — This class of wet compression presents two cases for consideration: the first, wherein all liquid carried into the compressing cylinder vaporizes at or before closure of the inlet valve; the second, wherein a portion remains unvaporized at that instant, and hence is present when the actual compression stroke begins. The first of these is, in fact, ordinary dry compression of vapor which has done part of its refrigerating work during admission to the cylinder instead of in the pipe coils.

It is a common practice in ammonia compression to permit the vapor to come to the machine wet. The evidence of wetness is the frosting of the suction pipe and often the bonnet of the suction valve. If vaporization be in progress all the way to the valve, it is fair to presume that more or less liquid is present at the valve and goes on into the cylinder. The vaporization of such liquid during admission takes heat from the metal parts which otherwise would superheat the vapor. The prac-

tice involves a little danger, under certain conditions, since there is a possibility that liquid may come to the machine as "slugs" of such weight that their vaporization may not be completed before the end of the expulsion stroke.

The result of an analysis of a typical case,* in comparison with compression of superheated vapor, is presented in the heat-work diagrams of Figure 43. For this we take the case of ammonia vapor coming to the compressor at 30 lbs. pressure, absolute, and saturation temperature, and assume that during inflow to cylinder it is heated to 40° temperature, then com-
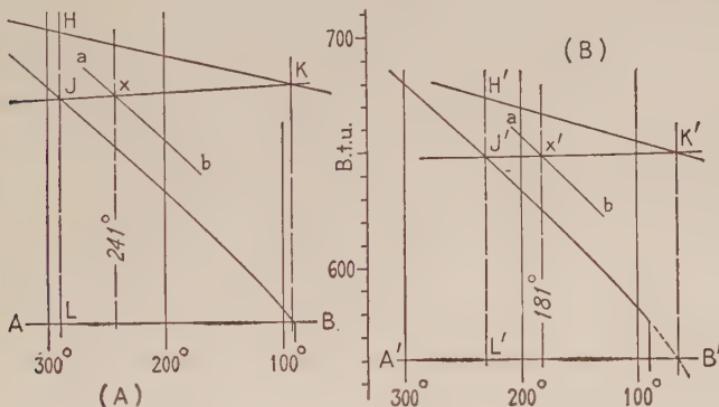
FIG. 43.   COMPARATIVE HEAT-WORK DIAGRAMS.   SUPERHEATED
AND SATURATED AMMONIA VAPOR

pressed to 180 lbs. pressure. Neglecting any probable pressure-drop at the inlet valve, the diagram is normal, as shown at (A). The heat taken from metal parts during the inflow is that which raises the vapor temperature from that of saturation, or − 0.6°, up to 40°, being the difference in heat contents at the two temperatures, or $634.6 − 611.6 = 23$ B.t.u. per pound. Neglecting machine friction and assuming that there are no other heat losses or gains, we lay off the curve a–b at uniform distance 23 B.t.u. above the curve for final internal energy, and its intersection with line $J−K$ falls on the actual final temperature vertical, that for 241°, at which the net total work per pound of ammonia scales 119.6 B.t.u., heat equivalent. The

power required per ton of refrigeration, without allowance for
general condition and friction factors, is

$$\frac{119.6 \times .4271}{42.45} = 1.203 \; H.\,P.$$

The latent heat of vaporization of ammonia at 30 lbs. pres-
sure being 569.3 B.t.u. per lb. (Bureau of Standards Tables),
the quality of the vapor which carries 23 B.t.u. less than its full
latent heat capacity is

$$\frac{569.3 - 23}{569.3} = .9596.$$

The weight of ammonia necessary per ton of refrigeration with
vapor leaving the coils of this quality is .449 lb.   The heat-
work diagram is that shown at (B), compression beginning
on saturated vapor.   The heat of compression passing tempo-
rarily to metal and from thence to the incoming ammonia is
23 B.t.u. as before, determining the final temperature by
intersection point x' at 181°, at which the net total work per
lb. is 107 B.t.u.   The required power per ton of refrigeration
for this case is

$$\frac{107 \times .449}{42.45} = 1.132 \; H.P.$$

In this second case the contra-work by incoming vapor might
at first glance appear to be only that done by 0.9596 lb. of
ammonia instead of that done by a full pound.   However, the
other 0.0404 lb. does its share of work when it vaporizes,
whether outside or inside of the cylinder, provided that the
vaporization takes place prior to the finish of the admission
stroke of the piston; hence the entire pound is effective.

For a third case with wet vapor we shall assume that more
liquid is carried into the cylinder than can be vaporized before
closure of the inlet valve.   Let the quality as before passing
that valve be 90 per cent, and let heat reception capacity
remain 23 B.t.u. as in the preceding cases.   Now the 23 B.t.u.
of heat vaporizes the same weight of ammonia as before,
.0404 lb.; therefore the work of admission is that done by

.90 + .0404 = .9404 *lb.* of ammonia, which corresponds to the quality of the vapor on which compression begins, 94.04 per cent. The initial internal energy as for vapor of this quality is 529.3 B.t.u. per lb. (from Figure 6). Using this as the base, we construct the heat-work diagram of Figure 44. The final internal energy is that of a full pound of vapor. Points *J* and *H* are determined in the usual manner, using differences between initial and final heat contents as read from Figures 5 and 6. Point *K* calls for special treatment, as follows:

The initial *PV* product is that of the volume of .9404 lb. of
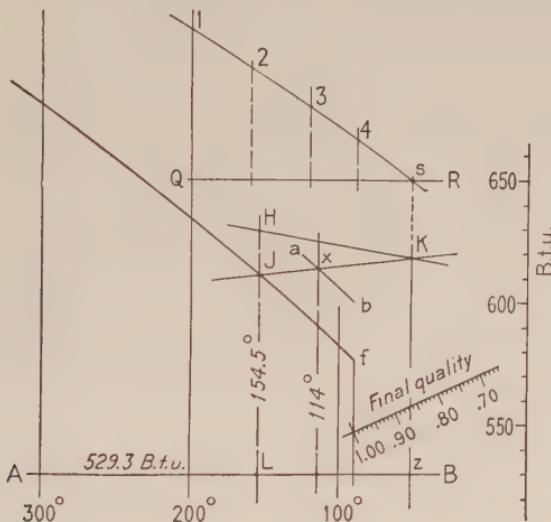
FIG. 44.   COMPRESSION OF WET VAPOR

vapor multiplied by the pressure, 30 lbs., and this must equal the product of the volume of a whole pound as at final pressure multiplied by that pressure. While one might make use of the isocaloric point as in previous cases, it is better here to use the true *Iso-PV* point, found by a simple artifice. Lay off the horizontal line *Q–R* at a height corresponding to the initial volume, measured above any convenient base or datum line, the latter not being shown in the figure. Then from the same reference line lay off the volumes per pound at the final pressure for a series of temperatures, say at 200°, 160°, 120°, and

at saturation, thus determining location of the points 1, 2, 3, 4, etc., and through these points draw a smooth curve to intersect Q–R at s. Through point s erect the Iso-PV vertical, and thereon measure K–z as read from Figure 21.

At the final temperature revealed by the diagram, 114°, the net total work per pound scales 94.5 B.t.u., but of every pound circulated .10 lb. has been cooled in the coils without itself doing any useful work therein, and the weight of ammonia required per ton of refrigeration mounts to .4862 lb. The required power per ton, then, is

$$\frac{94.5 \times .4862}{42.45} = 1.082 \; H.P.$$

With wet vapor or wet compression, it is possible to cool the compressor cylinder to such temperature that the compressed vapor shall still contain moisture in suspension. The quality of such discharged vapor may be found by the simple artifice of laying a decimal scale across the diagram in such position that the mark for 1.00 shall fall on the condensation temperature vertical while line K–z is at the quality computed for that point. Thus, in the example in hand the initial volume per lb. (volume of 30-lb., 94.04 per cent quality vapor) is 8.69 cu. ft.; and hence the volume per lb. of 180-lb. vapor at the Iso-PV point must be

$$\frac{8.69 \times 30}{180} = 1.447 \; cubic \; feet.$$

The volume of a pound of 180–lb. saturated vapor being 1.667 cu. ft., the quality corresponding to volume 1.447 is 1.447/1.667 = .868; and we lay the scale at such inclination as brings this graduation on the line K–z. Now we may scale the net total work in the usual manner for any final quality between .868 and 1.000; readings taken to the right of the Iso-PV vertical may not be trustworthy.

Wet Compression. — The last example, where a quantity of liquid was found in the cylinder at beginning of compression, illustrated one class of true wet compression. Nearly or quite

the same result is attained in machines in which a measured quantity of liquid is injected into the compressor after closure of the inlet valve. This latter practice is not attended with the same danger as the former, since the weight of liquid injected is subject to reasonably close control by the operator. If before injection the liquid be cooled down to the initial temperature of compression, we have nearly the same case as that last discussed. Generally, however, the injected liquid is warm, for it is brought direct from the liquid receiver at approximately the condensation temperature.

When liquid ammonia is sprayed into the limited volume afforded by the cylinder, it passes from a region of high pressure into one of low pressure and immediately begins to vaporize, taking such heat as required from adjacent substances or from the body of vapor found in the cylinder. In actual compression it is hardly to be presumed that any considerable change in quality can thus occur before the compression stroke begins, but we shall take an extreme case and assume that there is time for completion of whatever change might ensue. For this case let 0.9 lb. of dry, 30-lb. ammonia vapor enter the cylinder and be heated during the inflow to 40°, thereby taking $23 \times .9 = 20.7$ *B.t.u.* of heat from the cylinder metal, while 0.1 lb. of liquid at 89.8° temperature and 180 lbs. pressure is injected or sprayed into the cylinder just after the inlet valve has closed. The volume of the 40° vapor being 10.2 cu. ft. per lb., the 0.9 lb. occupies $10.2 \times .9 = 9.18$ *cu. ft.* in the cylinder. We ignore the volume occupied by the liquid, because it is too small to affect the result materially. The combined initial total heat content is

$$vapor\ 634.6 \times .9 = 571.1\ B.t.u.,$$
$$liquid\ 143.3 \times .1 = \underline{\ 14.3}\ B.t.u.$$
$$\overline{585.4}\ B.t.u.$$

Here we have two governing conditions: heat content, 585.4 B.t.u., and volume, 9.18 cu. ft.; and it is desired to know the quality and pressure of the vapor which can fill this volume and retain this heat content. The problem calls for

the use of a Mollier chart that includes curves for volume, omitted from Figure 5. By locating in the Bureau of Standards chart the point where the curve for volume 9.18 intersects that for heat content, 585.4, we find it to be at pressure 28.7 lbs. and quality 95.4 per cent. (Even with the large-scale chart, several trial readings may be necessary to obtain results which check throughout.) This means that compression starts on vapor of 95.4 per cent quality and 28.7 lbs. pressure. Compressing to 180 lbs., we draw the heat-work diagram, (D) of Figure 45, and mark the line a–b at 20.7 B.t.u. above G–f. The intersection of a–b with J–K determines the final tem-
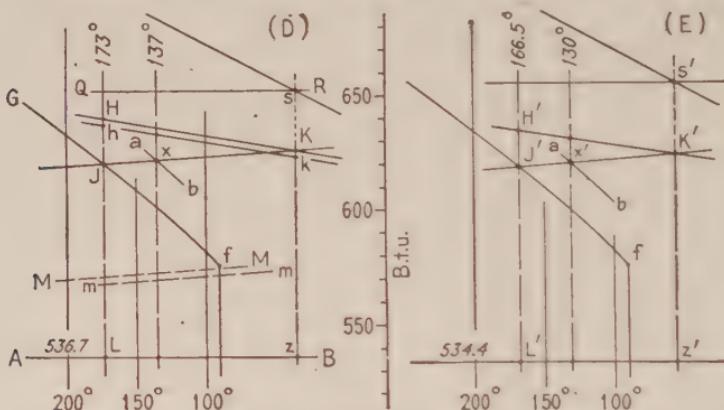
FIG. 45. WET COMPRESSION OF AMMONIA VAPOR

perature, 137°. A correction must be applied before scaling the work as at this or any temperature, as follows:

The contra-work shown by the diagram as drawn is that which would be done by 0.954 lb. of vapor at 28.7 lbs. pressure, its volume being 9.18 cu. ft. Converted into B.t.u. of heat equivalent this is

$$9.18 \times 28.7 \times \frac{144}{777.5} = 48.8 \ B.t.u.$$

The actual contra-work is that done by the 0.9 lb. of 30-lb. vapor which fills the cylinder volume 9.18 cu. ft., and this has the value

$$9.18 \times 30 \times \frac{144}{777.5} = 51.1 \ B.t.u.$$

The line $M–M$, drawn at constant distance 48.8 B.t.u. below $J–K$, marks the normal contra-work; but as the actual contra-work is 51.1 B.t.u., we draw line $m–m$ at 51.1 B.t.u. below $J–K$, the difference being 2.3 B.t.u., and likewise the resultant net total work curve $h–k$ at 2.3 B.t.u. below $H–K$. Measuring to this, the work as at 137° final temperature is found to be 97 B.t.u. per pound of ammonia. This determines the power per ton of refrigeration, thus

$$Power = \frac{97 \times .4271}{42.45 \times .90} = 1.084 \ H.P.$$

In the case just considered the initial partial vaporization of liquid and its fall in pressure were accompanied by a change in entropy. The entropy of .90 lb. of 40°, 30-lb. vapor is 1.2461, while that of 0.10 lb. of 89.8° liquid is .0295, the total being 1.2756; but the entropy of the resultant vapor of 95.4 per cent quality and at 28.7 lbs. pressure is found by simple computation, or from the chart, to be 1.2827, and this is the entropy used to find the adiabatic temperature for the heat-work diagram. It is hardly possible that the sprayed liquid could fully vaporize during compression without fall in pressure, accompanied by change in entropy; however, just as we assumed in the preceding case that the quality and pressure became stabilized before compression began, for another extreme case we shall now assume that vaporization of the injected liquid may be completed during compression without change in entropy or initial pressure. Starting, then, with 30 lbs. and entropy 1.2756, we find the temperature as after adiabatic compression to be 166.5°, the complete diagram being (E) of Figure 45. In this case the actual contra-work is that which governs in the diagram construction, and hence no correction is needed. The required power per ton of refrigeration is the same as in preceding case, as the work at the final temperature, 130°, measures 97 B.t.u. as in that case. With other pressures and qualities there might be a small difference between work values, in which case the true value would lie somewhere between. The normal diagram (E) serves all practical purposes.

External Work of Ammonia Vapor. — In cases like those of Figure 45 where it is necessary to make use of the external work value for a pound or fraction of a pound of ammonia vapor, it is readily found by reading the heat contents from
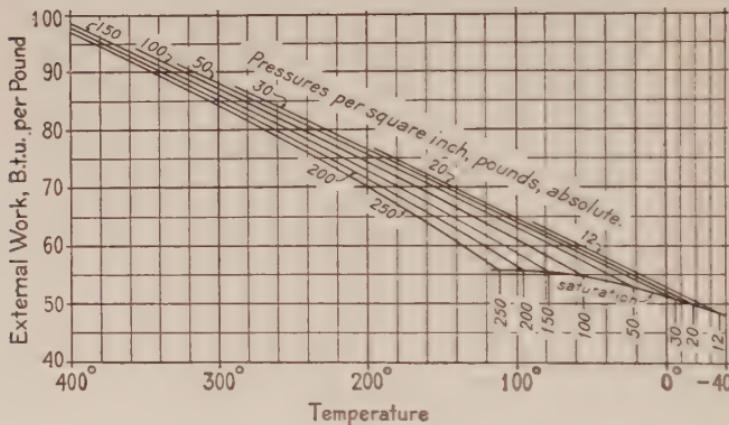
FIG. 46.   CHART SHOWING EXTERNAL WORK PER POUND OF
SATURATED OR SUPERHEATED AMMONIA VAPOR

Figures 5 and 6 and taking their difference.   For example, the external work for one pound of 30-lb., 40° vapor, is the difference between 634.6 and 577.8 B.t.u., or 56.8 B.t.u.   It is convenient, however, to have at hand a chart constructed to show such differences; Figure 46 is such a chart, covering the range of pressures and temperatures dealt with in refrigeration work.

CHAPTER XI

DISPLACEMENT AND COMPRESSION OF GAS BY GAS

Displacement of Gas by Gas, General Case. — In the discussions of Figures 25 and 29, it was shown that the heat equivalent of the work of expulsion or of exhaust is in any case, for any final temperature, equivalent to the difference between the total heat and the internal energy contents of the
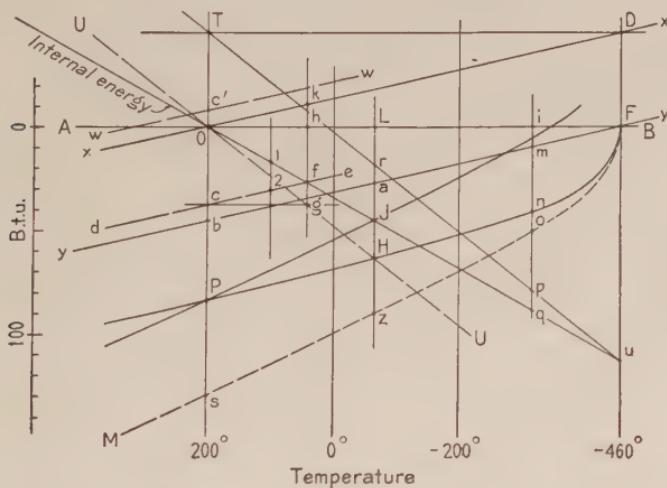
FIG. 47.   GENERAL GRAPH.   DISPLACEMENT OF GAS BY GAS

gas as at that temperature.   For convenience a portion of the general graph of Figure 25 has been redrawn as Figure 47. In this we add the straight line y–y, parallel to x–x, making O–$b = T$–O, the vertical measurement from y–y to A–B at any temperature being equal to the vertical measurement between final heat lines.   That is,

$$O\text{–}b = T\text{–}O = P\text{–}s; \quad L\text{–}a = r\text{–}j = H\text{–}z; \quad i\text{–}m = p\text{–}q = n\text{–}o; \text{ etc.}$$

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Suppose, now, that a pound of air at 88.2 lbs. absolute pressure enters a cylinder and, without any expansion having taken place, is allowed by the opening of the exhaust valve to pass out into and mingle with external air which is at 14.7 lbs. pressure. Here the pressure falls abruptly from 88.2 lbs. to 14.7 lbs., but the volume within the cylinder is constant during the fall; therefore, if the product of initial pressure and volume be PV, the product of the final pressure and the volume of the air in the cylinder is necessarily $\dfrac{PV}{6}$. The heat equivalent of the initial PV product being O–b, if b–c be one-sixth of O–b it represents the actual work of exhaust. Stating the fact otherwise, if the external work, O–b, be represented by X, and the ratio of the pressure as at close of admission (or at end of expansion, in the case of an expanding vapor) to the actual exhaust pressure be represented by r, we may write

$$Quantity \ \ O\text{–}c = \frac{X \ (r - 1)}{r}.$$

This holds true for negative work as well. For instance, if the exhaust should be against an absolute pressure of 102.9 lbs. per square inch, then we should have $r = 0.8571$ and

$$Quantity \ \ O\text{–}c' = \frac{X \ (0.8571 - 1)}{0.8571} = -0.1667 \, X.$$

The value represented by X, the external work, may easily be computed. It is the product of the absolute temperature multiplied by the difference between specific heat values. For air at 200° temperature it is

$$(200 + 460) \times (C_p - C_v) = 660 \times 0.0685 = 45.21 \ B.t.u. \ per \ lb.$$

Work of Displacement by Exhaust Gas. — Again using the example stated, when the exhaust valve is opened and the pressure falls from 88.2 lbs. per square inch down to 14.7 lbs., five-sixths of the confined air flows out through the valve unassisted by any expenditure of mechanical energy by the machine, displacing a certain volume of air outside in order to

make room therein for itself. Figure 48 may illustrate the facts. Let the cylinder of (A) contain the one pound of air at 88.2 lbs. pressure, the area of the piston (weight) being one square foot, thus making the total external pressure 12,700 lbs., under which the volume of the air is 2.77 cu. ft. We shall ignore friction, assuming that the weight is free to rise without frictional resistance. Now let five-sixths of the pressure (the weight shown) be removed, that is, let it become that of the outer atmosphere, 14.7 lbs. per square inch, or 2,117 lbs. on the piston area. The pressure thus being reduced, with no expenditure of energy by the confined air, the latter then at
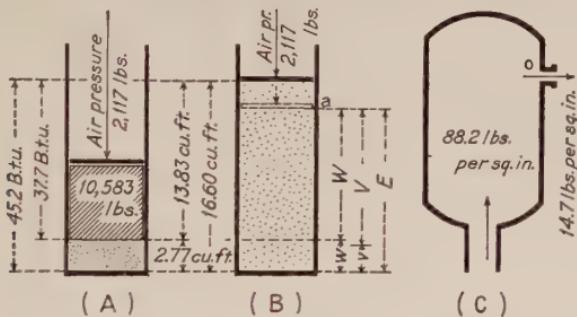
FIG. 48. DISPLACEMENT WORK. COMPARISON OF EX-
HAUST DISPLACEMENT WITH FLOW FROM AN ORIFICE

once proceeds to expand to a volume which shall be consistent with the reduced pressure. Should the temperature remain constant at 200° throughout the expansion (not possible unless heat be received from without), then the expanded volume would be six times the initial, or 16.60 cubic feet, the increase being exactly equal to the volume of external air displaced, 13.83 cubic feet.

Displacement of outside air in the manner indicated requires the expenditure of a portion of the store of internal energy carried by the expanding air; consequently the temperature of the latter must fall with that loss unless the heat spent be replaced by a like quantity of heat coming into it from some outside source of supply. Any decrease in the temperature is accompanied by shrinkage in volume; hence the colder the

expanding air becomes, the smaller the volume of external air displaced thereby.   If it displace 13.83 cu. ft., as for 200° final temperature, then the heat equivalent of the work done in displacement is

$$13.83 \times 14.7 \times \frac{144}{777.5} = 37.7 \ B.t.u.$$

Suppose, now, that the expanding air receives less heat from outside sources than it expends in displacement work, its temperature in consequence falling to something lower than 200° and its final volume being less than 16.60 cu. ft.   For instance, let the piston face rise only to point a, at which the volume is that marked E, of which V is five-sixths and v is one-sixth.   The volume of external air displaced is equivalent to that shown as W, not V, the initial volume being constant.   It is evident that any values of W correspond to external work done in expansion.   For instance, if temperature be maintained at 200°, the work of displacement, in heat equivalent, is five-sixths of the total external work value, or 37.7 B.t.u. as already stated.   Likewise, the work of displacing any volume W is the difference between the external work at an unknown temperature corresponding to volume E and the external work corresponding to volume w, the latter being constant and having a known value, one-sixth of 45.2 B.t.u., or 7.5 B.t.u.   Returning now to Figure 47, we had located point c, then and now found to be 7.5 B.t.u. above point b.   Draw line d–e through c parallel to y–y to intersection with the final internal energy line at f, also draw the horizontal line c–g.   Now we have

$$f\text{–}k = O\text{–}c = g\text{–}h, \text{ and } f\text{–}h = (O\text{–}c) - (f\text{–}g).$$

Now f–h is the heat equivalent of the energy spent by the air in displacement work, and at the same time it is the measure of the difference between the internal energy of air at 200° temperature and that at the temperature determined by f. The latter temperature, therefore, is the final temperature as after expansion unless there be loss or gain of heat during the expansion.   It is a sort of adiabatic expansion without change of pressure and without loss or gain of heat to or from outside

substances, but with change of temperature and with change of entropy and heat content.

Should heat be received from without, the final temperature would be higher than represented by the vertical through f. For example, should the quantity 1–2 be received, either from the external air being displaced or from neighboring metal or other substance, then the final temperature of the exhaust would be 100°. Should there be a loss instead of a gain the
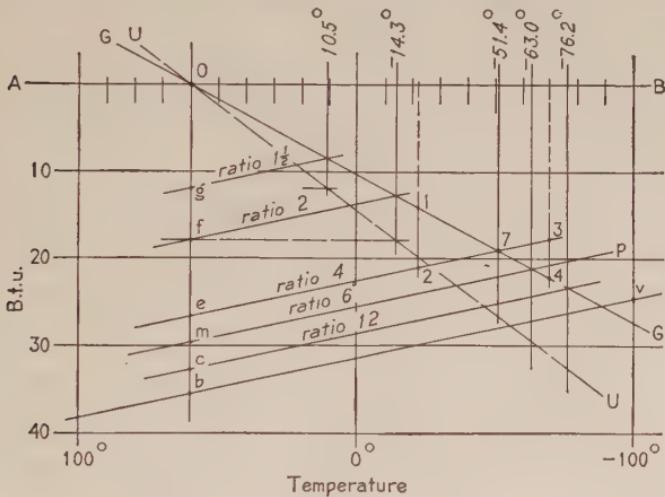
FIG. 49. HEAT-WORK DIAGRAMS. DISPLACEMENT OF GAS
BY GAS

temperature would fall to something lower than that determined by f.

Heat-Work Diagram, Displacement of Gas by Gas. — Instead of the elaborate diagram of Figure 47, a very simple form suffices to find the theoretical temperature to which the exhaust air or gas falls. (See Figure 49.) The base line, A–B, is drawn in the usual manner, and likewise the temperature scale. On the initial temperature vertical, for 60° let us say, lay off from A–B the distance O–b, corresponding to the external work for air at 60°, that is, 35.6 B.t.u.; and on the vertical for some low temperature, say − 100°, lay off likewise the external work as for that temperature, determining the

point v. Then draw a straight line through those two points, this line to correspond to a portion of y–y of Figure 47. Next draw the line G–G for final internal energy, also the line U–U as used in Figures 25 and 29. Thus far the construction is independent of pressures and pressure ratios. Now, if the pressure of the exhaust air drop to, say, one-sixth of the pressure as at the instant when the exhaust port opened, locate point m at five-sixths of O–b from O; through it draw m–p parallel to b–v, and at its point of intersection with G–G read the final temperature, about $-63°$. If the expansion ratio were 4 instead of 6, the quantity O–e would be three-fourths of O–b, and the final temperature would be determined by point 7 as $-51.4°$. If the expansion ratio were 12, O–c would be made $11/12$ of O–b, and the final temperature would be $-76.2°$. In general terms, if the initial external work quantity O–b be represented by X, the ratio of first to second pressure be represented by r, and the distance of the governing point (c, or f, or m, etc.) below O by N, we may write

$$N = \frac{X\,(r - 1)}{r}. \tag{19}$$

Then, for example, with a ratio of $r = 1\frac{1}{2}$, the heat quantity represented by O–g is

$$N = O\text{–}g = \frac{35.6\,(1\frac{1}{2} - 1)}{1\frac{1}{2}} = 11.9 \; B.t.u.$$

Turning back to the discussion of Figure 47, we observe that the horizontal line drawn through point c intersects the final temperature vertical at point g which falls also on the line U–U drawn parallel to the line for final total heat, and it follows that the product of the temperature difference between points O and h multiplied by the specific heat of air, C_p, must be equivalent to O–c and be the change in total heat corresponding to the temperature change. That is, if the temperature as at h be $43°$, the fall in temperature is $200° - 43° = 157$ *degrees*, and $157 \times .2396 = 37.7 \; B.t.u.$ It follows, therefore, that we may write a general formula for finding what temperature

drop accompanies a decrease in pressure under such conditions as we have described, thus:

$$Drop\ in\ temperature = \frac{X\ (r-1)}{C_p r}. \tag{20}$$

In this formula

X = *the external work as at first (the higher) temperature,*

r = *ratio of first to fallen pressure, each absolute, and*

C_p = *specific heat at constant pressure, for the gas in question.*

Applying this formula to one of the cases treated by Figure 49, we take the ratio of pressures, r, as 4.   Then

$$Drop\ in\ temperature = \frac{35.6\ (4-1)}{.2396 \times 4} = 111.4\ degrees,$$

and $60° - 111.4° = -51.4°$, the final temperature as at the fallen pressure.

In the actual operation of an air engine or pneumatic tool using air at initial pressure throughout, exhausting at a lower pressure as above discussed, the final temperatures may not be expected to fall to those found by the formula or the heat-work diagram, for the reason that more or less heat must necessarily be absorbed from the metal of the engine or tool, the surrounding atmosphere, etc. Thus, again using Figure 49, if at pressure ratio 4 the quantity of heat marked by 1–2 should be received, the final temperature would be about $-22°$. On the other hand, if it were possible for the expanding air to part with the quantity marked by 3–4, the final temperature would fall to about $-70°$. This subject receives further mention in Chapters XIII and XV.

Neither the formula (20) nor the graphical method for finding the temperature drop applies to the case where the pressure drop is negative, that is, where the change is to a higher instead of to a lower pressure. That involves compression of one portion of gas or air by another portion, the latter of indefinite weight, a case treated in later paragraphs. Referring to Figure 47, if the actual back-pressure be 102.9 lbs. (ratio 6/7), the work of exhaust as done by the machine is truly represented

by the vertical distance between y–y and w–w, parallel lines, the latter drawn through point c' located by use of formula (19); but this work is in fact the work of forcing out the one pound of air plus whatever weight of air may have entered upon it from without when the exhaust valve opened. The internal energy line of the figure is that for one pound, whereas the actual weight of air expelled is indefinite; hence no value
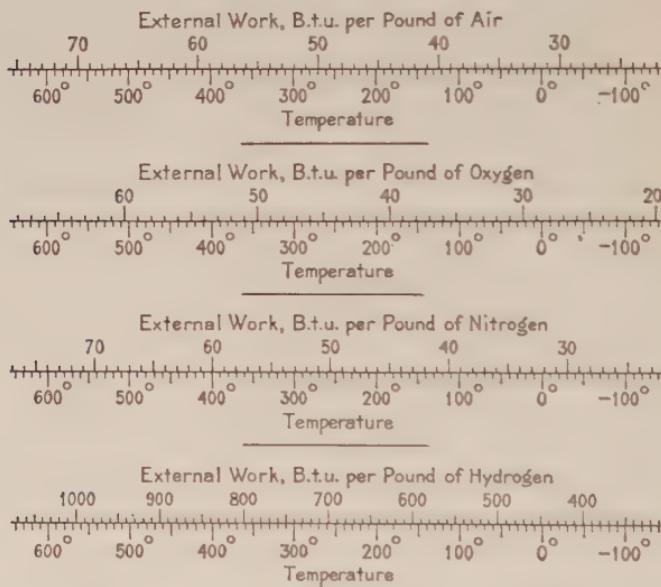
FIG. 50. EXTERNAL WORK OF AIR AND GASES

attaches to the point of intersection of w–w with the final internal energy line as plotted.

External Work of Gases. — The external work value for air or for any of the so-called perfect gases being independent of pressure, it is possible to represent it by a simple comparative scale. Figure 50 comprises four scales of this kind, for air, oxygen, nitrogen, and hydrogen, respectively. To use one, simply find the temperature on the lower scale, and then read the external work value as at that temperature at the corresponding point on the upper scale. As an example, for nitrogen

gas at 140° temperature, the external work per pound is
41.9 B.t.u.

Flow of Gas from an Orifice or a Nozzle. — Sketch (C) of
Figure 48 represents a container filled with air at 88.2 lbs.
absolute pressure, the container having an outlet orifice or a
nozzle-shaped discharge opening, *o*, out of which air flows in a
continuous stream into the external atmosphere where the pres-
sure is 14.7 lbs. per square inch.   Again the pressure-drop is
five-sixths of the initial pressure, just as in the case represented
at (A) and (B) and, likewise, all air flowing out must displace
external air in order to make place for itself therein.   In the
case last considered the expanding air was separated by a valve
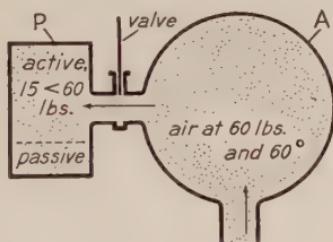
FIG. 51.   COMPRESSION OF GAS BY GAS

from the supply of high-pressure air; but here it is in communi-
cation with the high-pressure air supply, and the pressure of
the latter is maintained constant by means of a distant com-
pressor.   A discussion of the laws governing the flow of gases
from orifices or nozzles of varying size, length, contour, etc.,
and of work done in overcoming frictional and external re-
sistance to flow, heat of friction, heat lost and heat regained,
would be outside the scope of this work.   A diagram of the
order of Figure 47 or 49 is not applicable to the case of con-
tinuous flow.   The purpose of this scant mention of the subject
is to differentiate that case from those to which it does apply.

Compression of a Gas by Gas. — Let two containers be
arranged as shown in Figure 51, the one marked *A* being
maintained full of air at 60 lbs. absolute pressure and 60° tem-
perature and connecting by a valved passage to the other con-

tainer or receiver, P, the latter filled as at starting with air at 15 lbs. pressure and 80° temperature. For convenience we shall call this body of air found in P the passive air and that in A the active air.

Let the valve between the two containers be opened suddenly, to permit air to flow from A into P. We shall assume that the large valve area thus afforded permits the air to flow without any frictional resistance. In the case of flow from an orifice into the external air (Figure 48) the volume of the latter, unconfined, is so very great that its pressure cannot possibly be affected by reception of the small quantity of air flowing out from the orifice; but in the case now being considered, we have but a small quantity of air in receiver P, and hence not a particle of air may enter from A without causing a rise in the receiver pressure. The second increment of active air compresses not only the passive air but the first increment of active air as well, and so on until in due time the pressure in P becomes equal to that in A and at that instant the flow ceases. Every particle of active air that finds place for its self-enlarged volume in P must in like manner reduce the volume of whatever air it finds in P. This is compression, accompanied by transformation of the compressing energy into heat exactly as in the better-known case of compression of gas or vapor by a metal piston. The passive air is compressed from initial 15 lbs. to final 60 lbs. pressure; that is, in the ratio 1/4. Each and every particle of active air entering the receiver is re-compressed, in ratio varying for successive particles all the way from 1 4 to 1 1, each particle from an initial pressure slightly higher than that of its predecessor. All of this compression is accompanied by rise in temperature. The compression may or may not be adiabatic; if it be adiabatic then all energy used for compression and for re-compression remains as heat in the combined passive and active air.

Every particle of air flowing from A into the low-pressure container P experiences a temporary drop in temperature, because of the use of a part of its internal energy for self-expansion. We need not now investigate the extent of this

varying temperature change. The force acting to compress air in receiver P (all friction neglected) is the exact equivalent of the external work value of whatever weight of active air that may pass from A to P. It cannot be anything else. This being true, we need not be concerned with the intermediate conditions in the receiver; we are interested in the final temperature, by whatever path attained.

If all of the heat of compression remain in the compressed air and if no heat be received by it from any other source, this
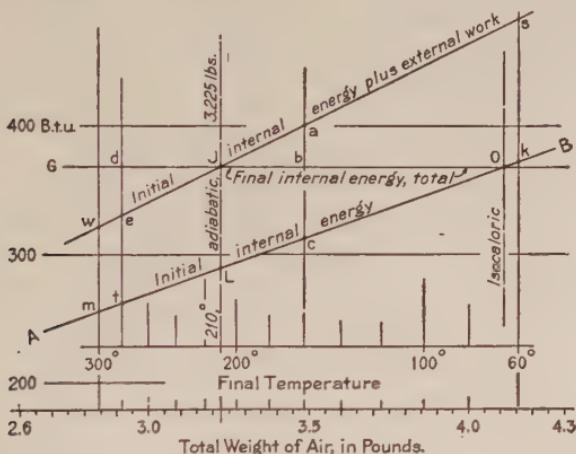
FIG. 52. HEAT-WORK DIAGRAM, COMPRESSION OF AIR
BY AIR

being the adiabatic case, then the internal energy of the air as found in the receiver P at end of the flow from A is necessarily the sum of the initial internal energies of the passive and the active air plus the external work of the active air. Not knowing the final temperature, we do not know the weight of the active air which may have passed from A into P. We have two unknown quantities. However, both may be found by use of another variant of the heat-work diagram.

Heat-Work Diagram, Compression of Gas by Gas. — First, Figure 52, a scale for weight is to be laid down in place of the temperature scale of the normal heat-work diagram. For convenience we let the weight of the passive air be one pound.

The volume required to contain this pound of air when at 80°
temperature and 15 lbs. pressure is found in Table IV to be
13.32 cubic feet; this is therefore the capacity of receiver P.
Now the volume occupied by a pound of air if at 60° tempera-
ture and 60 lbs. pressure is 3.207 cubic feet; hence if P should
contain air at that temperature and pressure, its weight would
be 13.32/3.207 = 4.154 *lbs.*, and in our case 1.00 lb. of that
weight would be the passive air and the remaining 3.154 lbs.
the active air which had entered the receiver. Similarly, we
compute the weight that would be required to fill receiver P
if at 60 lbs. pressure and some higher temperature, say at
300°, finding it for the latter to be 2.842 lbs., of which the
active portion would be 1.842 lbs. Our weight scale, we thus
have found, should cover the range between 4.154 and 2.842,
as drawn.

Next, and similarly, we find the weight of air which would
be contained in receiver P if at each of a series of temperatures
between 60° and 300°, at convenient intervals, in this manner
determining the locations of a sufficient number of points to
permit the construction of a temperature scale covering the
range from 300° to 60°. For instance, at 220° the required
weight is 3.177 lbs., and we erect the vertical for 220° through
the point 3.177 of the weight scale.

Next we compute the internal energy content of one pound
of air as at 80° (the passive air) and find it to be 92.4 B.t.u.,
reckoned above 0.0 at absolute zero as the datum. To this
we add the internal energy of 3.154 lbs. of (active) air at 60°,
the sum being 373.0 B.t.u.; and this quantity we lay off on the
vertical marking 60° temperature and 4.154 lbs. weight, meas-
uring it upward from some convenient horizontal base line
(not shown in the Figure), thus determining the location of
point k. Similarly, the internal energy of 1.842 lbs. of air at
60° is 163.9 B.t.u.; and this added to the 92.4 B.t.u. of the
pound of passive air gives the internal energy for the 2.842 lbs.
as 256.3 B.t.u., which, measured on the line for that weight,
from the same base as in first instance, determines the location
of the point m. We then draw a straight line through points

k and m, which shall serve as an initial internal energy base line, corresponding in its uses to base line $A-B$ of the normal heat-work diagram. In this case, however, the base is for a variable weight of air, whereas the base in the normal diagram is for one pound (or other convenient unit weight) of the air, gas, or vapor for which it is used.

Proceeding, we find the external work as at 60° temperature for 3.154 lbs. and for 1.842 lbs. of active air, the respective quantities being 112.3 and 65.6 B.t.u., and these laid off vertically above points k and m determine the locations of points s and w, through which we draw a straight line that shall mark, for any weight of air, the sum of the internal energies of that weight of passive and active air combined plus the external work value of the active air alone and at the initial 60-degree temperature. As the next step, we compute the internal energy contents as at several final temperatures and corresponding weights of air, as for 2.842 lbs. at 300°, for 2.919 lbs. at 280°, for 3.000 lbs. at 260°, etc. The computations will yield identical results in every instance, 369.6 B.t.u., and we draw a horizontal line, $G-O$, at this distance (measured in heat units) above the datum we had selected.

At the adiabatic temperature the work done in compression, which can be nothing else but the equivalent of the external work of whatever weight of active air be required at that temperature, is equal to the gain in internal energy for the total weight of active and passive air; and therefore the intersection of the lines at J marks the adiabatic temperature, in this case 210°, at which the total weight of air scales as 3.22 or 3.23 lbs. Final temperature cannot be higher, then, than 210° unless heat be received from some outside source. In order to attain a final temperature of 280°, for example, the heat quantity measured by $e-d$ would have to be received from an outside source. On the other hand, if the final temperature be lower than 210°, it is because heat has passed from the confined air to some other substance. For example, at 160° the heat thus lost is that measured at $a-b$.

Inasmuch as all lines of our diagram are straight lines, it is

easy to check by simple proportion the results obtained by the graphical method. The values thus computed are 3.225 lbs. for the weight and 209.7° for temperature.

Quantities such as e–d and a–b of the diagram measure either the deficiency or the waste of external work for the weight of active air passing into the receiver P. For instance, again using temperature 280°, the quantity measured by t–d is, by computation, 106.47 B.t.u., while the quantity t–e, for 1.919 lbs. of active air as required at that temperature, is 68.35 B.t.u. The difference, 38.12 B.t.u., necessarily is furnished by heat coming into the air from the container metal or other foreign source. By a similar computation one may find the deficiency or waste as at any other final temperature and verify the quantity read therefor from the heat-work diagram.

Heat-Work Diagram, Compression of Vapor by Vapor. — This case is quite similar to the preceding, but here the total final internal energy for passive and active vapor is not a constant quantity; hence the line or curve therefor is not a straight horizontal line as it was in Figure 52. Figure 53 is an example for a case with vapor of ammonia. The passive vapor is at 12 lbs., absolute, at saturation temperature and of 98 per cent quality, while the active vapor is at 30 lbs., saturation temperature, and of 96 per cent quality.

For this case we first lay down the weight scale as before. The initial volume of a pound of the passive vapor is found to be 21.33 cu. ft. (using the Ammonia Tables), and this divided by the tabulated volume per pound as at 30 lbs. pressure and any final temperature gives as the quotient the total weight capacity for that temperature. In this way we find a series of weights which determine the locations of a sufficient number of points to permit construction of a temperature scale. Then we proceed substantially as described for the case with air, using heat values from the Ammonia Tables or from Figures 5 and 6, and volumes as read from the tables. The adiabatic temperature is that at the intersection point J, about 16.4°, at which the total weight of the ammonia is, by scale, 2.212 lbs., meaning that 1.212 lbs. of active vapor has entered

upon and compressed the passive vapor. If heat quantity a–b were to pass to outside substances, the final temperature would be brought to 10°, at which the weight of the active vapor
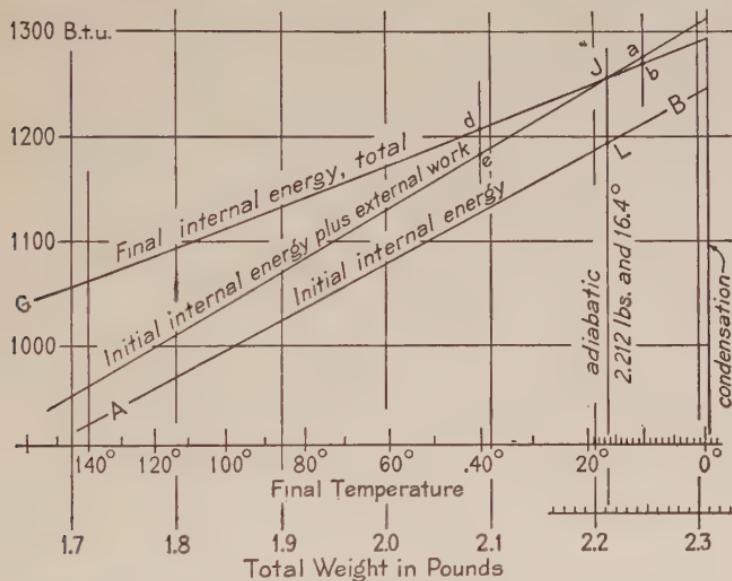
FIG. 53. HEAT-WORK DIAGRAM, COMPRESSION OF VAPOR
BY VAPOR
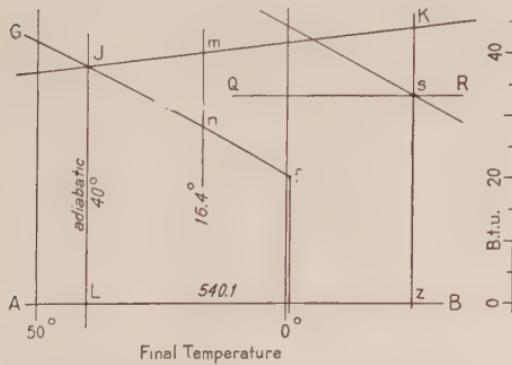
FIG. 54. WORK OF COMPRESSION BY VAPOR

would be 1.245 lbs. If, instead of losing heat, the quantity d–e were to be received, then the final temperature would be 40° and the weight of active vapor 1.088 lbs., etc. The accu-

racy of the readings for any temperature may be verified by computation of final volumes, weights, and heat contents.

Figure 53 shows the work of compression as at the adiabatic temperature to be 62 B.t.u., this being the external work for the 1.212 lbs. of active vapor used. If we now make a simple heat-work diagram for the compression of one pound of 98 per cent quality ammonia vapor from 12 lbs. to 30 lbs. pressure, as in Figure 54, we shall find that the work required as at 16.4° final temperature is only 40 B.t.u. That is, the expenditure of that quantity of energy on a piston in a metal compressor would have brought the one pound of passive vapor to the same pressure and temperature as it has been brought by the expenditure of 62 B.t.u. of potential energy during the process of compression by another body of vapor. Of the 40 B.t.u. actually required for the compression, 12 B.t.u., measured by $m-n$, is returned as heat to the active vapor.

CHAPTER XII

COMPRESSION FROM MODIFIED BACK–PRESSURE

Initial Pressure May Differ from Back-Pressure. — In normal compression the actual work begins on gas found at same pressure as that at which contra-work was done by it during its entrance to the cylinder. It is possible, by various means, to modify the initial cylinder pressure. (See Chapter X.) A better-known case is that where a valved passage connects the cylinder with a chamber or pipe containing gas at pressure either higher or lower than the back-pressure: if the valve be opened as the piston nears the end of its suction stroke, then closed before it covers any considerable part of the compression stroke, the pressure inside the cylinder tends to equalize with that in the connecting chamber or pipe, and compression begins on gas at the changed pressure.

Referring to Figure 10, the contra-work by back-pressure is measured by the vertical distance between curves *C–D* and *M–F* and is a constant quantity determined by initial temperature (and by initial pressure, in the case of vapor). The work of expulsion is measured by the vertical distance between curves *E–F* and *M–F*. It is a variable dependent on final temperature, or rather on final volume which varies as the final (absolute) temperature varies. Final temperature is influenced by initial heat, ratio of compression, and heat loss or gain; but the actual back-pressure may be varied without affecting any of these factors. Neither the work of compression nor of expulsion is affected by any change in back-pressure as long as the initial pressure is held constant, but the net total work is the sum of these minus the contra-work, and hence is increased or diminished by exactly the same quantity that contra-work is diminished or increased. In Figure 10, with a decrease in back-pressure, curve *M–F* rises throughout or falls with increase in back-pressure.

Multiple Effect Compression. — Conditions frequently occur in refrigeration work that seem to call for a compressor designed to operate with modified back-pressure. As an instance, in a cold-storage plant where there are a few rooms used as freezers and held at temperatures very low in comparison with those that suffice for other rooms, if there be but one standard compressor in use, it must take the refrigerant vapor (ammonia, we assume) from all room coils at the low pressure and temperature necessary for the coldest; thus the back-pressure might have to be maintained as low as 0 lbs. on the gage, whereas a pressure of perhaps 20 to 25 lbs. might be amply low to yield the temperatures required in the warmer rooms. If the service for the latter forms a considerable part of the total (it may be the greater part) it is clear that such one-pressure operation entails needless daily expense incident to the high compression ratio used throughout. An obvious solution of this problem is the use of two machines, one working with the higher back-pressure, the other with the lower; this adds to the duties of the attendants, however, and the chances for trouble in operation and at the same time leaves one machine to work at the objectionable high compression ratio. To meet these conditions, a compressor arranged to receive ammonia at both pressures may be the proper choice.

Figure 55 illustrates the principle involved in the type of machine under discussion. It represents a single-acting vertical compressor cylinder, the three views showing the piston at different points in its travel. No effort has been made to show actual details of construction, nor do the sketches represent any particular make of machine. At (A) the piston is on down-stroke, and vapor at the lower of the two suction pressures is flowing through the inlet valve of piston into the compression chamber. As the piston nears the lower end of stroke, it uncovers a number of ports leading from the ring-shaped manifold *H*, which contains vapor at a pressure higher than that of the vapor which entered through valve *S* and already fills the compression chamber. In the interval between uncovering and closing of the ports from *H*, the vapor therefrom flows into the

cylinder until the pressure therein becomes equal to that in manifold H, Sketch (B). (This is the theory; in practice the pressure attained may not quite equal that in H.) Here we have a case in compression of vapor by vapor, as discussed in the preceding chapter. Actual compression by machine begins the instant that the ports from H are fully covered by the piston in its up-stroke. Discharge is through valve D, Sketch (C), not different from that of the standard compressor.

Suppose that the lower suction pressure, which is the back-pressure proper, is 12 lbs. absolute per square inch, while that
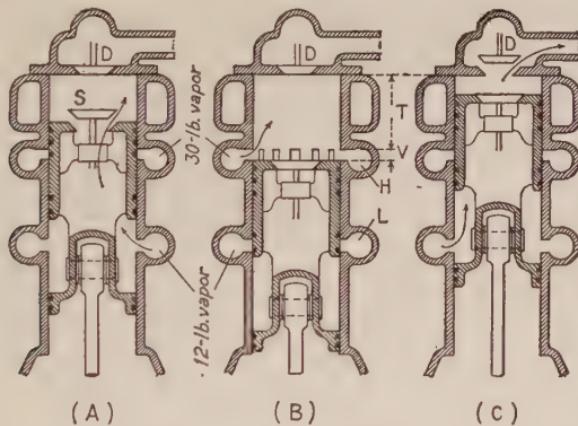
FIG. 55. COMPRESSION FROM MODIFIED INITIAL PRESSURE

in the manifold is 30 lbs. No contra-work against the piston is done by the 30-lb. vapor. Whatever effective work may be done by it during its admission is work of compression on the 12-lb. vapor it finds in the cylinder. Whatever contra-work there be is done by the fraction of the cylinder charge that enters through the suction valve at 12 lbs. pressure. Here we have a case of compression from modified back-pressure, with more than normal net total work required, meaning, of course, in comparison with the work required for normal compression from the full 30 lbs. back-pressure.

Heat-Work Diagram for Multiple Effect Compression. — For convenience we shall take the same data as used in the preceding paragraph and for the example in connection with

Figure 53 of Chapter XI, where it was found that, if the vapor-
by-vapor compression were adiabatic, the final temperature
would be 16.4°, the weights of the passive and active vapors
having the ratio 1.000/1.212. Starting with vapor of that tem-
perature, let compression be to 200 lbs., absolute pressure. The
heat-work diagram is that shown in Figure 56. The normal part
of this, comprising points L, J, H, O and K and the several
curves and lines thereby determined, is constructed in the usual
manner, for one pound of vapor; but if we should measure
the net total work as at any temperature between curve $H-K$
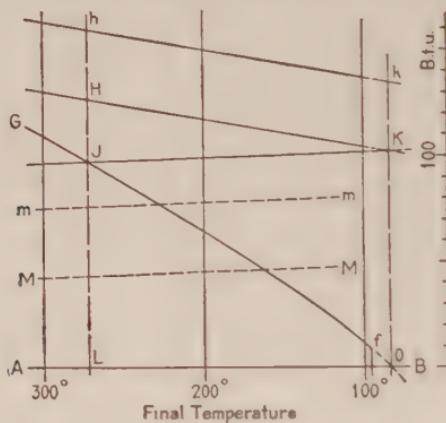
FIG. 56.   HEAT-WORK DIAGRAM, COMPRESSION FROM
MODIFIED INITIAL PRESSURE

and the base $A-B$, the result would be the work as it would be
with a normal back-pressure of 30 lbs. exerted throughout the
admission stroke. For that condition the normal contra-work
line would be $M-M$, at a uniform distance of 53.7 B.t.u. below
$J-K$, and the quantity 53.7 B.t.u. would be the external work
for one pound of 30-lb. vapor at 16.4° temperature. Now as a
matter of fact, the contra-work is done by only 1/2.212, or
0.452 lb., of 12-lb. vapor, and that is of 98 per cent quality. Since
the external work value per lb. for saturated vapor is 48.4 B.t.u.,
the actual contra-work done on the piston per pound of am-
monia compressed is $48.4 \times 0.98 \times 0.452 = 21.4$ $B.t.u.$, in heat
equivalent. This is 32.3 B.t.u. less than the 53.7 B.t.u. of

contra-work shown by $M-M$; hence we must raise the net total work curve $H-K$ a distance of 32.3 B.t.u. to a new and correct position, as at $h-k$, the work as at any final temperature then being scalable from $A-B$ to this curve (straight line) in the usual manner.

The adiabatic discharge temperature as found for construction of Figure 56 is moderately high, and this with compression from an initial temperature of only 16.4°. It is obvious that had one or both vapors come to the compressor superheated through a few degrees, instead of wet as we assumed, the final temperature of their mixture would have been higher than 16.4°. Again referring to Figure 53, we find the included angle $d-J-e$ to be so acute that even a comparatively small change in the relative position of the two lines will be reflected in a very considerable change in the position of their intersection, the adiabatic point. The temperature zone involved is lower than that which can be affected by jacket water; hence the practicable method for attaining a low initial temperature for the piston compression is to let the vapors come to the machine carrying a little liquid in suspension.

CHAPTER XIII

EXPANSION OF AIR OR GAS TO MODIFIED EXHAUST PRESSURE

Air Expansion in Tools and Engines. — It has been shown that the maximum of work obtainable from a pound of expanding gas or vapor is obtained when the expansion is adiabatic. It is seldom practicable to attain this in an ordinary air engine, for the reason that heat is almost certain to pass either to or from the air being expanded. Any heat received raises the final temperature and increases by a little the net work done, the measurement in the heat-work diagram being made to the left of the adiabatic; however, any heat thus received is (generally) heat that has been lost by conduction from the warm compressed air supply, and it has been shown (page 99) that heat thus taken from the supply and used for cylinder heating is of less value than if it had been retained and held the supply at higher temperature. Any heat lost during expansion lowers the final temperature and decreases the work done. Much depends on the initial temperature and the ratio of expansion. If the initial temperature be high and the ratio small, the adiabatic temperature may be near that of the external atmosphere and parts of the machine, with probable loss of heat during expansion and final temperature lower than the adiabatic. On the other hand, with low initial temperatures and even moderate ratios of expansion, the adiabatic is far below atmospheric and machine temperatures. Hence more or less heat flows to the air being expanded.

In the actual operation of air engines and tools, it often is impossible to maintain a high temperature for the air supply; and in spite of such aid as may be given by heat from machine parts and external atmosphere, the exhaust temperature may be so low as to cause serious trouble from freezing of moisture

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in the exhaust passages, valves, etc.   To avoid such trouble, the air may be expanded in lower ratio to a pressure above that of the atmosphere into which it is to exhaust, or even be used at full pressure throughout, with no expansion, as in pneumatic tools.   Again, there may be cases where the air expands in the cylinder to a pressure lower than that of the outside atmosphere into which it must exhaust.   There are, then, four cases to be considered, viz.:

(a) *Operation with full initial pressure, no expansion;*

(b) *Operation with final pressure (after expansion) higher than that of exhaust, known as partial or incomplete expansion;*

(c) *Operation with final pressure equal to that of exhaust, this being the normal case, complete expansion; and,*

(d) *Operation with final pressure lower than that of exhaust, or over-expansion.*

(a) **Full-Pressure Operation.** — In this case the air remains at its initial pressure for the entire travel of the piston (or very nearly) and then on opening of a valve falls quite abruptly to that of the exhaust.   The net total work done is that of admission minus that for expulsion (exhaust).   The problems involved have been dealt with in the discussions of Figures 47, 48, and 49, the heat-work diagram being very simple.   For an example to serve in all four cases, let us take air at 100 lbs. absolute pressure and 240° temperature, and let the exhaust pressure for all be 20 lbs.   With full-pressure operation, no expansion, the ratio of final (same as initial) to exhaust pressure is therefore 5.   The heat-work diagram is that shown as (A) of Figure 57, which is of the same order as Figure 49.   The external work for air at 240° is 48 B.t.u.   Then, using formula (19) of Chapter XI, we have

$$N = \frac{X(r-1)}{r}; \quad \text{or,} \quad N = \frac{48(5-1)}{5} = 38.4 \ B.t.u.,$$

and this is laid off as *O–c* on the diagram.   This is the work done in the engine or tool, per pound of air used, regardless of what the actual exhaust temperature may be.   Drawing

c–f parallel to y–y to intersect the final internal energy line at f, we find that the adiabatic exhaust temperature would be 80°. However, heat is almost certain to pass to or from the air during the exhaust. If it should pass to the air, in quantity represented by 6–7, the final temperature would be whatever
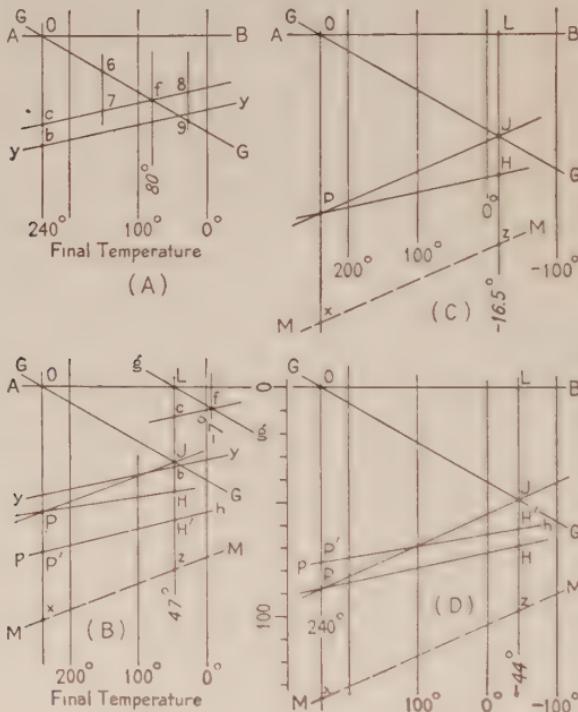
FIG. 57. EXPANSION OF AIR. HEAT-WORK DIAGRAMS FOR FULL-PRESSURE OPERATION, PARTIAL EXPANSION, COMPLETE EXPANSION AND OVER-EXPANSION

that determines, about 150°; if it should pass from the air, in quantity measured by 8–9, let us say, then the exhaust temperature would be about 25°, etc.

Conversely, if the exhaust temperature be 150°, it shows that the heat quantity 6–7 is being received from the cylinder metal or (and) the atmosphere being displaced. The work of displacement of external atmosphere varies with final exhaust temperature, as was discussed in Chapter XI, but the useful

work done by the air against the piston is not affected by such variations; it remains constant at 38.4 B.t.u. as long as the temperature as at instant admission ceases remains 240° and the ratio 5. As a check on the final exhaust temperature we may apply formula (20), thus:

$$Fall\ in\ temperature = \frac{X\ (r-1)}{C_p r} = \frac{48\ (5-1)}{.2396 \times 5} = 160.1°$$

The exhaust temperature, then, is 240° − 160.1° = 79.9°.

(b) **Partial or Incomplete Expansion.** — For this case let the final pressure be 32 lbs., absolute, the exhaust pressure again being 20 lbs. The expansion ratio is now 100/32 = 3.125, while the ratio of final pressure to that of exhaust is 32/20 = 1.6. The heat-work diagram is (B) of Figure 57. The temperature of the air as at the instant expansion ceases, if adiabatic, is 47° and the pressure 32 lbs. Next we lay off the contra-work line, M–M, as in the general graph of Figure 25. If the exhaust work were normal, it would be 48 B.t.u. at 240° and 34.7 B.t.u. at 47°; and these quantities laid off from M–M would determine the locations of points P and H of the normal diagram; but as a matter of fact, the exhaust work now is only the external work minus that found by formula (19) or

$$Exhaust\ work\ at\ 240° = 48 - \frac{48\ (1.6-1)}{1.6} = 30\ B.t.u.,$$

and

$$Exhaust\ work\ at\ 47° = 34.7 - \frac{34.7\ (1.6-1)}{1.6} = 21.7\ B.t.u.$$

The quantities 30 and 21.7 B.t.u. laid off above M–M at the respective temperatures determine the locations of the two points P' and H' of the correct net total work line, p–h. The work done by the air is measurable to this from A–B, at any final temperature.

Considering next the actual exhaust temperature, we draw line y–y for external work and then mark the point c, making b–$c = H'$–z. Draw g–g through L parallel to G–G and intersect it at f by the line c–f drawn through c parallel to y–y.

The point f falls on temperature $-7°$. Therefore, if both the expansion and the exhaust be adiabatic, the final temperature of the latter is $-7°$. In practice the exhaust air takes more or less heat from the metal of the machine and from the atmosphere into which it is forced. For instance, if it should thus take heat equivalent to $L-c$, the final temperature would be $47°$.

However, reception or loss of heat may occur during expansion as well as during exhaust; and thus we have two variables, either or both of which may be loss or gain, and the case is still further complicated by the item of frictional heating; hence final temperature is not a reliable index to net total work. As has been shown, heat changes during exhaust do not affect the work done by the machine; they merely alter the displacement work that the exhaust air must do in making room for itself outside the cylinder. In the case of compression, it was concluded that discharge temperature might be taken as the final temperature of compression without introducing any great error; unfortunately, in the case of partial or over-expansion, the change of temperature during exhaust is so great that final temperature by thermometer does not give with any accuracy the value needed for the heat-work diagram, the temperature as at the end of expansion.

(c) **Complete Expansion.** — This is the normal case, expansion being continued down to the exhaust pressure, neither more nor less. To attain this condition in practice, very accurate adjustment of the point of cut-off is required. The heat-work diagram is normal, similar to Figure 24. In our example it is (C) of Figure 57. Since contra-work line is $M-M$, the exhaust work at any temperatur is that measured between $M-M$ and $P-H$; that is, the resultant net total work line falls on $P-H$ and not below it as in (B) or above it as in (D). The expansion ratio is $100/20 = 5$. The exhaust pressure ratio is $20/20 = 1.00$. Applying formula (19) to this limiting case, we have (the external work of air at $-16.5°$ being 30.4 B.t.u.)

$$Exhaust\ work\ at -16.5° = 30.4 - \frac{30.4\ (1-1)}{1} = 30.4\ B.t.u.$$

That is, points c and L coincide, showing that the exhaust air is not called upon to use any of its own energy for displacing external air; all displacement work is done by the machine. However, the air may gain or lose heat during the exhaust stroke, as in any case.

(d) **Over-Expansion.** — This happens in an engine cylinder when the cut-off occurs too early in the stroke. Let us assume that the expansion is to 16 lbs. pressure, absolute. The expansion ratio is therefore $100/16 = 6.25$, while the exhaust ratio is $16/20 = 0.8$. The heat-work diagram is that shown as (D) of
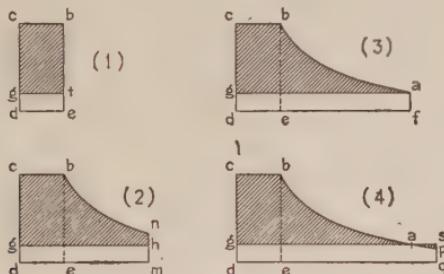
FIG. 58. NON-CLEARANCE INDICATOR DIAGRAMS FOR FULL-PRESSURE OPERATION, PARTIAL, COMPLETE, AND OVER-EXPANSION

Figure 57, the temperature after adiabatic expansion being $-44°$. At the latter the work of exhaust is

$$Work\ of\ exhaust = 28.5 - \frac{28.5\ (.8 - 1.0)}{.8} = 35.6\ B.t.u.,$$

and similarly, at $240°$ it is 60.0 B.t.u. Using these values we establish point H' by making $z-H' = 35.6\ B.t.u.$, and P' by making $x-P' = 60\ B.t.u.$, thus determining the position of the true net total work line, $p-h$, to which the work is to be measured.

When the exhaust valve opens, referring to case (D) of Figure 57, the air from the exhaust pipe or manifold comes in upon the air found in the cylinder and compresses it from pressure 16 lbs. to 20 lbs., which is a case of compression of air by air, described in detail in Chapter XII. Here we cannot use the simple

graphical method of Figure 49 to find final temperature, nor formula (20) for temperature change. Moreover, the volume of the active or compressing body of air is so large and indefinite, relative to that of the air being compressed, that the temperature rise in the mixture must remain an unknown quantity.

The indicator diagrams for the four cases of expansion here considered have the general outlines of those shown in Figure 58, which differ from actual indicator cards in that they do not show any curve for re-compression into a cushioning space and have sharp corners instead of the rounded corners traced by an indicator pencil. For full-pressure operation the diagram is of the order of Sketch (1), the net work done being that shown by the shaded area. For partial expansion the diagram takes the form at (2) with abrupt pressure drop at end of expansion and exhaust at a lower pressure. The normal diagram is that of (3), while with over-expansion there is a loop at the end of the expansion curve as at (4), the net work being the difference between that measured by the shaded area a–b–c–g and the negative work measured by area a–s–p.

CHAPTER XIV

COMPOUND AND MULTI-STAGE COMPRESSION

Stage Compression with Intercooling. — In all cases of compression thus far discussed, the change from initial to final pressure has been accomplished in one cylinder. Instead of this it may be done in two, three, four, or more cylinders, each receiving the gas at or a little below the pressure at which it was exhausted from the preceding cylinder. This is compound or multi-stage compression; compound if there be two stages, multi-stage if three or more.

In any compressor the cylinder surface exposed to cooling water is relatively small; and as the cylinder metal is quite thick, the transfer of heat from gas to jacket water is slow and incomplete; hence the gas when discharged retains the major part of the heat of compression. With two or more stages, the gas while in transit from one cylinder to the next may be cooled efficiently in a chamber or series of pipe coils known as an intercooler, so that any cylinder will receive gas at a temperature differing but little from that at which it entered the preceding cylinder. Very high discharge temperatures, inevitable with high compression ratios, are objectionable and impracticable for several reasons; and where the compression must be to a very high pressure the use of a compound or multi-stage machine, with intercooling, is imperative. Such machines have long been in common use for air and gases and are being introduced with success in ammonia refrigeration work.

Heat-Work Diagram for Compound Compression. — A diagram is required for each compression stage; and if the true initial and final conditions for that stage be known or obtainable by computation, the diagrams involve no unusual features. A few examples will be sufficient. First we take a case in three-stage compression of air, from initial temperature 80° and

atmospheric pressure to a final pressure of 18 atmospheres. The compression ratio for first cylinder is 3; for the second, 3; and for the third, 2. The product of these stage ratios is 18. For comparison we also make a heat-work diagram for compression in one stage, ratio 18. For this latter the temperature as after adiabatic compression would be 772°, at which the net total work per lb. of air would be 165.8 B.t.u., as measured at $H-L$ of Figure 59. This high final temperature is above the ignition point of lubricating oils, and further, the thick metal
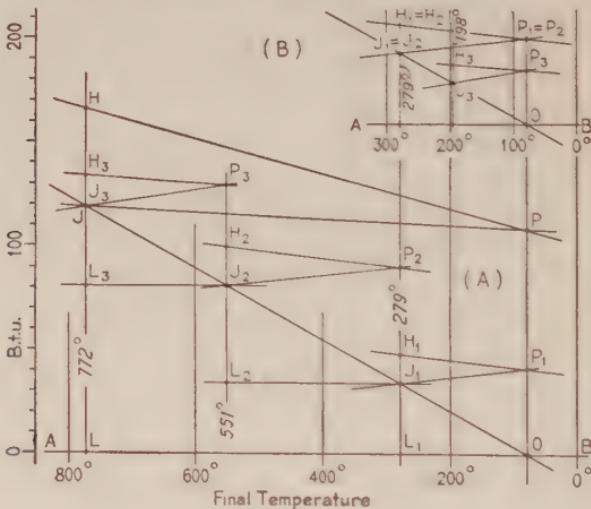
FIG. 59.   THREE-STAGE COMPRESSION WITHOUT
INTERCOOLING

called for by the high final pressure would preclude any possibility of efficient cooling by water in a cylinder jacket; hence this single-stage compression is wholly impracticable. In any case the final temperature should be brought below the flashpoint of the cylinder oil used.

Now if the compression be done in three stages, with the specified ratios 3, 3, and 2, the first or low-pressure cylinder will discharge at a pressure of 3 atmospheres and temperature 279° (if adiabatic), the net total work being 47.7 B.t.u. as at H_1-L_1. (In this example we neglect frictional heating, inflow heating, and pressure-drop at valves, for simplicity.)   If there

be no intercooler between first and second cylinders, the second will receive its charge at 279° and in turn will discharge (if adiabatic) at 551° and 9 atmospheres pressure, the work being 65.2 B.t.u. as at H_2-L_2. Finally, the third cylinder will receive at 551° and discharge at 772° and 18 atmospheres pressure, its net total work being 52.9 B.t.u. as at H_3-L_3. The final temperature is thus the same as after single-stage compression to same final pressure and the total work is the same; that is:

$$47.7 + 65.2 + 52.9 = 165.8 \; B.t.u.$$

Thus it is proved that multi-stage compression without intercooling effects no saving in power if the compression be adiabatic in every cylinder.

Again, suppose that in both the single-stage and the multi-stage machines the compression be isothermal. Now the heat equivalent of the work in single-stage is 107.2 B.t.u., as at $O-P$; that of the first cylinder in multi-stage is $O-P_1$, that is, 40.8 B.t.u., that of the second is the same, the compression ratios being equal, while that of the third is 25.6 B.t.u. This last is shown in the diagram at (B), in which have been redrawn the diagrams for first and second cylinders. Again the work for the three cylinders equals that for single-stage, that is,

$$40.8 + 40.8 + 25.6 = 107.2 \; B.t.u.$$

Finally, let us assume that all compression is adiabatic but with perfect intercooling between cylinders of the multi-stage machine. Again using (B) of Figure 59, we see that the work measures, respectively, 47.7, 47.7, and 28.3 B.t.u., the sum of these being 123.7 B.t.u., which is a little less than 75 per cent of the work shown for the impracticable single-stage compression, while the final temperature is only 198°, the highest for any stage being 279°. If a water-jacket were used on any or all cylinders, the work total would be made still less. Any number of combinations and diagrams might be made, using varying degrees of intercooling and varying efficiencies of water jackets, etc.

Intermediate Pressures in Multi-Stage Compression. — In the last example it was assumed that the intermediate pressures

were the same at any given stage in all cases considered, but that could not happen unless the sizes of the second and third cylinders were varied to suit the differing temperatures. In an actual compressor the temperatures are changeable, necessarily; and as the cylinder dimensions are fixed, the pressures must vary accordingly. A second example is given to bring out the facts in this connection. Let us suppose that we have a two-stage air compressor, receiving air at 60° and 15 lbs. and discharging at 135 lbs., absolute pressures. Let the first or low-pressure cylinder be of 12-inch bore and the high-pressure cylinder 7-inch bore, each 12-inch stroke and single-acting; and let a water-cooled intercooler be provided between cylinders. Further, let it be determined by actual experiment or by estimate that the temperature of the air rises 15 degrees during inflow, each cylinder; also that there is a drop in pressure of one pound per square inch at the inlet valve of low-pressure cylinder and of 2 lbs. at inlet to the other, while the thermometer indicates that the temperature of the air as it leaves the intercooler is 90°. Moreover, let it be known or assumed that the volumetric efficiency of the low-pressure cylinder, as affected by the factors of clearance and general condition, is 92 per cent and that of the high-pressure cylinder 90 per cent. The intercooler pressure is determined by these data and should agree with the result obtained by the following computation:

Briefly, we have

92 per cent of volume of l.p cylinder, 0.7226 cu. ft.
90 per cent of volume of h.p. cylinder, 0.2406 cu. ft.
Volume of one lb. of air, at 60° and 15 lbs., 12.82 cu. ft.
Volume of one lb. of air, at 75° and 14 lbs., 14.15 cu. ft.
Weight of air received per stroke, which must be the same for each cylinder, **0.7226/14.15 = 0.0511 lbs.**

The temperature of this weight of air after its entrance into the high-pressure cylinder is 90° + 15° = 105°, and we ascertain its pressure thus:

From page 12 we have the formula, $PV = WRT;$ or, if

working with pressures per square inch instead of square foot, it becomes $144pV = WRT$, and therefore, $p = \dfrac{WRT}{144V}$. Substituting in this the numerical values for our problem, we have

$$p = \frac{0.0511 \times 53.294 \times 565}{144 \times 0.2406} = 44.39\ lbs.,$$

the nominal pressure in the high-pressure cylinder at the instant the inlet valve thereto closes. It will be observed that this pressure varies with T, the absolute temperature. The higher the initial temperature in the high-pressure cylinder the higher the intercooler pressure, and vice versa. The high-pressure cylinder compresses air from 44.4 lbs. and 105° temperature to final pressure 135 lbs., the compression ratio thus being $135/44.4 = 3.04$, while the low-pressure cylinder takes air at 14 lbs. (in cylinder) and 75° temperature and compresses to $44.4 + 2.0 = 46.4$ *lbs.*, the compression ratio being 3.31.

Intercooler pressures may not be perfectly constant, as the volume of air contained is relatively small; and unless the inlet valve to the h.p. cylinder opens at same instant as the discharge from l.p. cylinder, and unless they likewise close together at a later instant, the intercooler pressure may rise or fall to some extent. The larger the intercooler volume, relative to one cylinder charge, the less the fluctuation in intercooler pressure. Passing over this matter for the moment and assuming constancy of intercooler pressure as found, we are now prepared to construct the heat-work diagram, Figure 60. While it might not be practicable to compress air in single-stage from 14 to 135 lbs., on account of excessive cylinder heating, a diagram for that case has been included for comparison, in which the air is presumed to heat to 90° during inflow and the pressure to drop to 14 lbs.

Comparing results as read in Figure 60, we see that much depends on the efficiency of the cylinder cooling in the several cases. Perhaps the very best that could be done with the single stage machine would be to bring its discharge temperature down to 350°, or thereabouts, and at that point the heat equivalent of

the net total work per lb. of air would be 103.4 B.t.u. In contrast, even if there were no water jacket on the low-pressure cylinder of the compound machine, its discharge temperature should hardly exceed 280° to 300°, with frictional heating taken into account; and with a water jacket might be brought down to perhaps 220° or lower. At 280°, the work measures 51.5 B.t.u.; at 220°, it measures 49.0 B.t.u. Similarly, the work for the second cylinder at 300° measures 49.5 B.t.u., or at 250° measures 48 B.t.u. The combined work as at the higher tem-
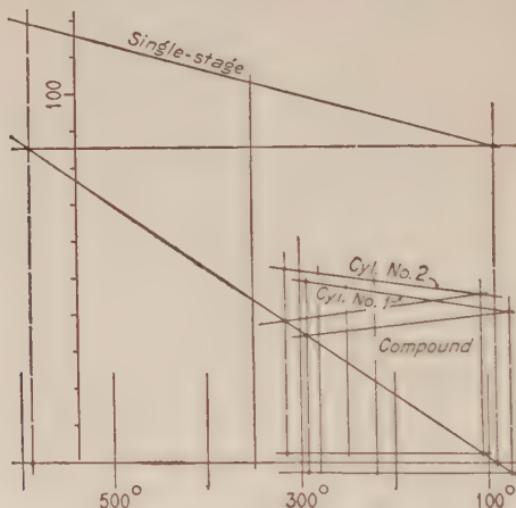
FIG. 60.   COMPOUND COMPRESSION WITH INTERCOOLING

peratures stated is 101 B.t.u., or 97 at the lower. Thus with only inefficient jacketing there is a saving as compared with the best possible jacketing and cold water supply for the single-stage machine, while the low final temperatures make lubrication a comparatively easy matter.

The computation shows that with cylinders of given size and efficiency the intercooler pressure is determined by the initial temperature in the following cylinder, or it is a near-truth to say that it is determined by the final temperature in the intercooler. The latter is influenced by the circulation of water through the intercooler itself and by the water made to flow through the jacket of the preceding cylinder.

Reverting now to the matter of fluctuations in intercooler pressure, we may say that any cylinder but the first may have to work with modified back-pressure, which was discussed in Chapter XII, also that any cylinder but the last may have to work with modified expulsion pressure. For instance in Figure 60, if the opening of the valve into the high-pressure cylinder be retarded, the pressure in intercooler must rise above that found by the computation. Thus, instead of discharging at 46.4 lbs., the low-pressure cylinder might have to discharge against a pressure of 50 lbs. or more during a part of its expulsion stroke, and in consequence the high-pressure cylinder would receive air at 50 lbs. or more at beginning of its admission stroke. This would require that the heat-work diagrams be modified, that for the low-pressure cylinder by raising the net work curve to show the increase in work of expulsion, that for the high-pressure cylinder by lowering the net work curve to show the increase in work of admission thereto. Conversely, the intercooler pressure might be abnormally low during part of the stroke, thus decreasing the work of first cylinder while increasing that of the second.

Compound Ammonia Compression. — When air or a gas is to be compressed, the initial temperature usually is as high at least as that of the atmosphere of the room wherein the compressor does its work, while the several inter-stage temperatures are such as may be controlled by use of cooling water of ordinary temperature. The conditions are very different for a compound ammonia compressor in a refrigerating plant, as it may be required to take vapor at temperature − 30° or lower. Moreover, the final discharge pressure is that necessary for condensation of the ammonia by the coldest water available for use on the condenser, and the coldest water available for use in an intercooler is at that same temperature. At any intermediate pressure the condensation temperature for the ammonia is far below that of the cooling water; hence the vapor must leave the intercooler retaining many degrees of superheat. For instance, if the pressure in intercooler be 40 lbs., absolute, a temperature of 11.7° would cause condensation to begin, and

it would be highly desirable to reduce the temperature of the vapor to within a few degrees of this, say to 20°; but if the coldest water for intercooler be at, say, 60°, then the tempera- ture of the ammonia hardly could be brought below 65°. Thus a water-cooled intercooler is able to take from the ammonia vapor only a portion of the heat of compression that it receives in the low-pressure cylinder. Even so, a saving may be effected in cases where compression must be from very low pressure and temperature; the direct saving in power may be small, but the lower discharge temperatures give a greater degree of freedom from lubrication troubles and evils caused by the presence of vaporized oil in the piping system.

It is not practicable to use the general formula, $PV = WRT$, for finding intermediate pressure in compound vapor compres- sion, because the quantity R for any vapor is a variable; but if tables of properties of the vapor be at hand, they serve the purpose. For example, suppose that the net capacity of the second cylinder be 0.4375 cu. ft., and that this is to be filled at each stroke by the known weight of ammonia received in first cylinder, let us say 0.0495 lb., and that the initial temperature in second cylinder is 100°. There is but one pressure at which all of these conditions are met, it being that at which the vol- ume of 100° vapor is 0.4375 0.0495 = 8.839 $cu. ft.$ per pound. Searching the ammonia tables, we find that this is at 39 lbs., absolute, and therefore the normal intercooler pressure is 39 lbs. plus whatever drop there may be between this and the discharge pressure of first cylinder.

Vapor-Cooled Intercooler. — The intercooler for ammonia vapor may be arranged for either complete or partial cooling by a diverted portion of the refrigerant liquid, this liquid being sprayed into the intercooler vapor content and taking therefrom the heat required for vaporization. With this arrangement the second cylinder must handle more ammonia than the first, differ- ing by the weight of the liquid sprayed; and an additional burden per pound of effective ammonia is put on the condenser. Doubtless the most economical arrangement provides for cooling by water to as low a temperature as attainable therewith,

followed by a further cooling by ammonia. For any given conditions the heat-work diagrams will show the work per pound, readily convertible to power per ton of refrigeration. Conditions are so varied that it is hardly safe to draw general conclusions; each case should be analyzed to determine whether or not the use of vapor-cooled apparatus is justified as a matter of economy.

CHAPTER XV

VAPOR EXPANSION

Expansion of Vapor without Condensation. — The heat-work diagram for this is similar to that for gas expansion. The only material difference is that for vapor it is necessary to make use of the *Iso-PV* temperature, or else the isocaloric, in place of the isothermal for determining the location of point K. For example see Figure 61, for steam, initial pressure, 180 lbs.; initial temperature, 773.1° (that is, with 400 degrees of superheat); and pressure after expansion, 20 lbs. The initial volume per pound is 4.09 cu. ft. (Marks and Davis's Tables.) By the steam tables, we find the *Iso-PV* temperature to be about 775°, differing but a little from the isothermal. The work at the *Iso-PV* temperature, arrived at by computation or by Figure 22, is 299 B.t.u. heat equivalent per pound of steam. Then making use of formulas (15), (11), and (12) to compute the work for several other final temperatures, including the adiabatic, we get data for plotting curves D–C and F–E. It is seen that these are nearly straight lines except in the vicinity of point P. Now if we draw the vertical for temperature through the isocaloric point O, and thereon mark the point K at the same level (work) as point P, it is found that the expansion and net work curves, if drawn through K instead of P, as straight lines, practically coincide with the plotted curves in the useful portion of the diagram, and therefore the use of K is preferable to that of P. Points J and H may be determined by use of the charts, Figures 7 and 8, or by use of steam tables, and having thus located points O, K, L, J, and H, the lines of the diagram are readily drawn. We have introduced the use of the formulas in this instance merely to show that straight lines instead of the work curves may be used without material error.

Again by Figure 61 we see that, although the steam carries 400 degrees of superheat at beginning of expansion, after its expansion to 20 lbs. pressure its temperature is but little above that at which condensation would begin. From a somewhat lower temperature, say 600°, either Figure 7 or 8 shows at once that the corresponding entropy curve cuts across the saturation curve, meaning that at the end of adiabatic expansion the steam is partially condensed to water, the final quality being readable
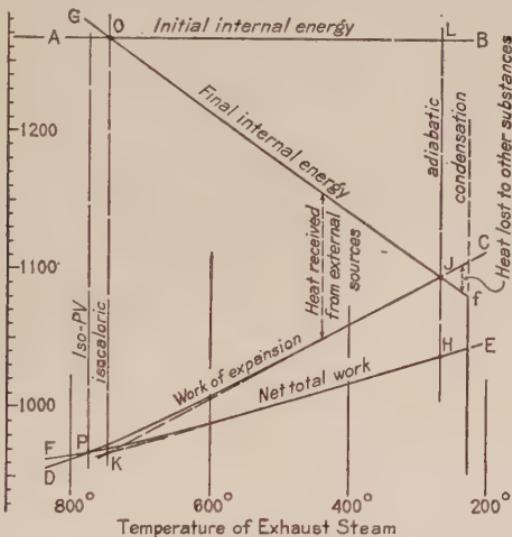
FIG. 61. HEAT-WORK DIAGRAM, EXPANSION OF STEAM WITHOUT CONDENSATION

from the chart. Expansion is thus accompanied by condensation in the ordinary steam engine. A modified or variant form of heat-work diagram is applicable, as discussed in the following paragraphs.

Expansion of Vapor with Condensation. — In Figures 25 and 29 the vertices of the expansion and net total work curves lie on the absolute zero temperature vertical, at which temperature the volume of the gas is supposed to be nil, in accord with the general law governing change in gas volumes, (Charles's law). Reduction in gas volume accompanies reduction in temperature. The lower limit for both is absolute zero. The fall in tempera-

ture is due to withdrawal of sensible heat from the gas. When a vapor condenses to liquid at constant pressure, the process is accompanied by decrease in volume but not by fall in temperature, the change of state and volume being due to withdrawal of latent heat of condensation from the vapor. The change in volume is relatively great. For instance, a pound of steam, saturated, at 20 lbs. absolute pressure and 228° temperature requires 20.08 cu. ft. of space; but when condensed to water having the same temperature, the volume required for the water is only 0.0168 cu. ft., roughly 1 1200 of the original volume. This is so small relative to the volume of the vapor that it may be neglected without material error, and we may say that the volume of the vapor becomes nil when the quality is made 0.0 per cent. **The volume of a gas, at constant pressure, varies directly as the absolute temperature; the volume of a wet vapor, at constant pressure, varies directly as the quality.**

General Graph for Expansion with Condensation. — It is possible to trace the curves for work of expansion and net total work of an expanding vapor through the entire range of final qualities, with a resulting graph quite similar to Figure 25. For an example let us take steam at initial pressure 140 lbs., absolute, and with 200 degrees of superheat, its temperature being 553.1°, and let the expansion be down to 20 lbs. pressure. The construction of the diagram, Figure 62, presents no new features to the left of the condensation temperature zone, the point K, the curve for final internal energy, that for final total heat and the base line $A–B$ for initial internal energy being drawn as in previous cases.

Suppose, now, that we use some convenient scale for final quality, as shown. Obviously, the heat content at quality 0.0 is the heat content of water at 228° temperature, about 196 B.t.u., and since there is but a fraction of a B.t.u. difference between the value for internal energy and that for total heat, one point may represent both, as point w; and then after drawing straight lines $w–f$ and $w–s$, we may scale the heat content as for any quality. This is simple enough, but we desire to locate the 0.0 volume vertical in such position that

the quality scale shall harmonize with the temperature scale and permit continuity of the work curves. To accomplish this, we select any convenient base line, say a–g; and from this lay off vertically to any convenient scale the volumes of 20-lb. vapor as at several temperatures, such as to* b for 628°, to d for 328°, to e at saturation, etc., and then through the points thus fixed draw a curve (practically a straight line) prolonged to intersection with a–g at t. Through t erect a vertical which is to represent absolute zero of volume at 0.0 per cent quality. This determines truly the position of the quality scale to which reference has been made. If instead of following this method, we should prolong the curves for final heat content and internal energy to their intersection, as at point u, we would determine the location of a point quite near the 0.0 vertical as found by the volume method; and that may be done if preferred or more convenient, but the volume method gives the more accurate location.

Using Figure 7 or 8, we find that the initial entropy is 1.692; and following this down to the line for 20 lbs. pressure, we see that the final quality after adiabatic expansion is 97.1 per cent, and we draw a vertical line at this point of our quality scale. The point where it cuts the line w–f is the adiabatic point, marked J, and L–J represents the difference between initial and final internal energy, as in any other heat-work diagram. Similarly we make L–H equal to the difference between initial and final total heat contents, and may then draw the lines K–J and K–H. Further, we make D–F equivalent to the initial external work and draw x–x to serve the same purpose as in Figure 29. The initial volume per pound is 4.24 cu. ft., and the quality of 20-lb. steam having this same volume would be $4.24/20.08 = 0.211$, which determines the location of the isovolumnic point, V. Then we make V–5 to equal m–V and find that we have four points in each work curve; that is, we have K, J, V, and D of the expansion curve and K, H, 5, and F of the net total work curve, and through these we draw the smooth curves as shown. The work is now scalable as for any quality of expanded vapor.

Ordinarily there is no necessity for showing the entire range of heat contents, and the diagram may take the form shown in Figure 63, which is for a case with initial pressure of the steam at 120 lbs.; initial temperature, 450°; final pressure,
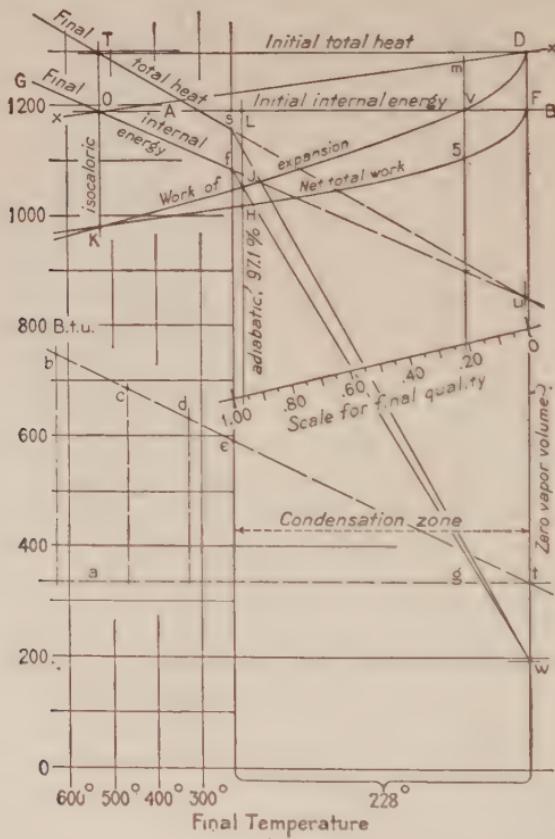
FIG. 62.   GENERAL GRAPH FOR EXPANSION OF STEAM,
WITH PARTIAL CONDENSATION

17 lbs.; and final quality (at adiabatic) 93.7 per cent. With the quality scale laid out as heretofore described, it is a simple matter to turn to Figure 8, read the energy content as for any conveniently low quality charted, 80 per cent for instance, and thus determine the location of a point, z, governing the location of straight line z–f, this serving the same purpose as w–f of Figure 62.

After a little practice, one can draw the work curves with surprising accuracy by use of the four-point method. The original drawing of Figure 63 was made in this manner, while computation of the work as for quality 60 per cent yields values marked by the small circles, their centers falling almost exactly on the curves previously drawn. Error is to be expected in the
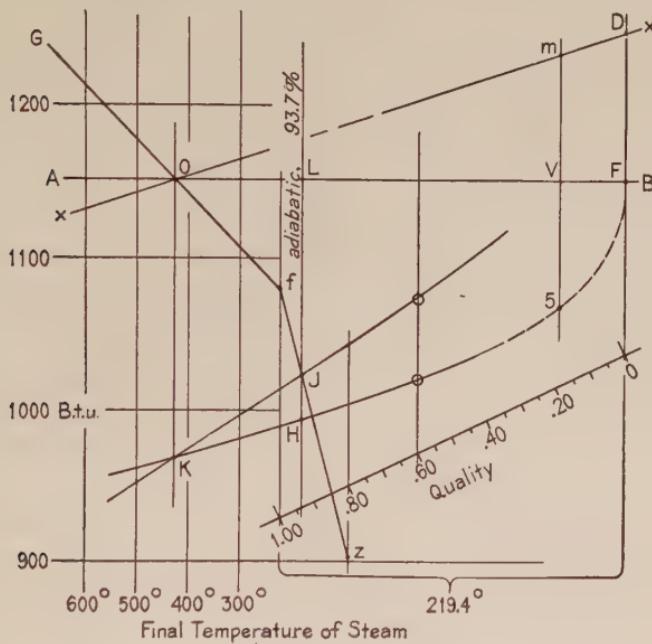
FIG. 63.   EXPANSION OF STEAM; TYPICAL HEAT-WORK
DIAGRAM

vicinity of point V, or beyond, but in practice we are only interested in values near to the adiabatic, where the lines are nearly straight.

In the construction of these diagrams it may be found that the vertical for the isocaloric temperature would lie to the right of the line for saturation temperature; and in such cases it is necessary to use the vertical for the true Iso-PV temperature or quality, which may fall to either the left or right of the saturation temperature vertical or coincide therewith. For

examples, suppose, first, that the initial quality of 26-lb. (absolute) steam be such as to make its volume 15 cu. ft. per lb., and that it is expanded down to 5 lbs. absolute pressure.   The initial PV volume is $26 \times 15 = 390$, and therefore the final volume must be

$$390/5 = 78 \ cu. \ ft. \ per \ lb.$$

Turning to the steam tables, we find that this corresponds to the volume of 5-lb. steam when carrying about 40 degrees of superheat, so the $Iso\text{-}PV$ vertical should be located in the diagram at 40 degrees to the left of the saturation temperature. For second example, suppose that the initial quality be such that initial volume is only 13.8 cu. ft. per lb. Now the volume of the final 5-lb. steam must be

$$26 \times 13.8/5 = 71.8 \ cu. \ ft.$$

As the volume of 5-lb. steam at saturation is 73.3 cu. ft. per lb. the quality as at the $Iso\text{-}PV$ point must be $71.8 \ 73.3 = 0.978$, *or* 97.8 *per cent*, at which point on the quality scale we should erect the vertical therefor. In cases like this we have no practical use for the part of the diagram which would lie to the left of the 100 per cent quality vertical, and hence that part may be omitted, as it has been from Figure 64, which is for steam with initial quality 92 per cent, absolute pressure 20 lbs., expanding down to 4 lbs. The process begins and ends in the wet vapor zone; hence the quality scale may be laid down at once and the curves drawn by the four-point method as in preceding cases.

External Work of Steam. — It is convenient, occasionally, to have a chart showing external work values for steam at ordinary working pressures and temperatures. Figure 65 is such a chart, similar in arrangement and purpose to Figure 46 for ammonia. Instead of using this, one may use the difference between readings from Figures 7 and 8, if preferred.

Modified Exhaust Pressure and Wet Steam. — The general case of expansion to a certain pressure, with exhaust against a lower pressure, was treated in Chapter XI, where it was seen that the work of displacement was of necessity accom-

panied by fall in temperature of the gas being expelled unless
the heat spent in displacement work was offset by an equal
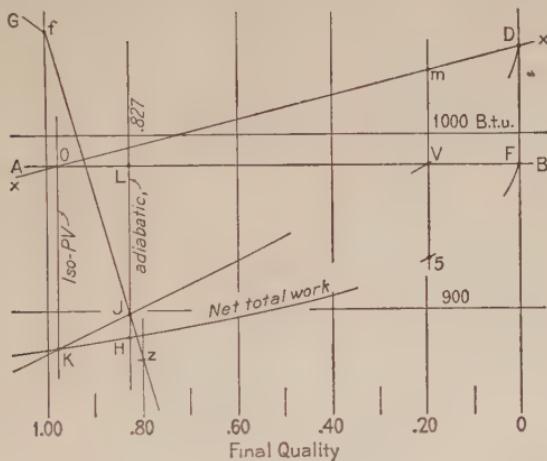
FIG. 64.   EXPANSION OF WET STEAM; HEAT-WORK
DIAGRAM
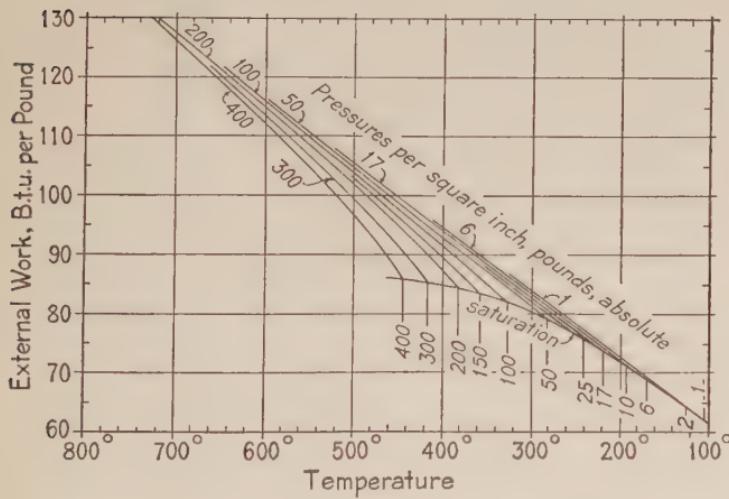
FIG. 65.   CHART SHOWING EXTERNAL WORK PER POUND
OF SATURATED OR SUPERHEATED STEAM

amount of heat from without.   The case with wet vapor is
quite similar, but here there is change in quality as well as in
temperature.   To illustrate, let us again use the data for which

Figure 63 was drawn, the final quality of 17-lb. steam having been found to be 93.7 per cent and its internal energy 1023 B.t.u. per pound.

We draw a simple diagram, Figure 66, starting with a quality scale and the known quality .937, and initial internal energy 1023 B.t.u. Next we find the external work for saturated steam at 17 lbs. pressure, determining thereby the location of point 1, also that for a wet steam, say of 80 per cent quality, the latter determining the location of point 2, each measured from $A-B$. Points 1 and 2 fix the position of the straight line
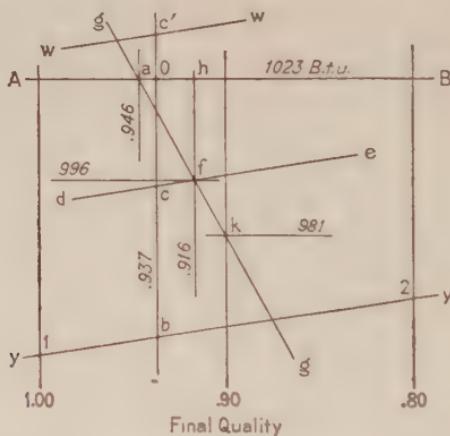
FIG. 66. CONDENSATION OF STEAM IN EXHAUST

$y-y$, similar to that in Figures 47 and 49. Suppose now that the actual external pressure against which the engine must exhaust be 10 lbs. instead of 17 lbs. per square inch. Then the actual work of exhaust is that measured at $c-b$ instead of the normal $O-b$, and $c-b$ is 10 17 of $O-b$. This work is constant, regardless of final quality. The steam which was at 17 lbs. pressure suddenly finds itself in a region of 10 lbs. pressure and proceeds to expand and make room for its low-pressure volume therein, necessarily doing work of displacement, with expenditure of energy accompanied by fall in temperature or change in quality. In the general case with gas or air, there was no available energy other than the sensible heat; but in this case

there is an available stock of heat latent as heat of vaporization, and this is called upon indirectly to do a share of the work of displacement.

Proceeding, we next lay down a line for internal energy of 10-lb. wet steam, which is readily done, as follows: Using Figure 8, find the quality of 10-lb. steam having 1023 B.t.u. internal energy content, it being .946, and mark this point at a. Then find the energy content for some other quality, say 90 per cent, which is 981.0 B.t.u., thereby fixing the location of point marked k. Now draw straight line g–g through a and k. Line g–g marks the internal energy content of 10-lb. vapor for the qualities embraced. Now the intersection of g–g with the line d–e drawn through c parallel to y–y, the point f, marks the final quality of the steam, 91.6 per cent. This method, it will be observed, is simply one of equating work of displacement to loss of internal energy. The heat energy used for displacement is in first instance sensible heat of the steam, this being replaced in part by latent heat set free by condensation.

If we were dealing with over-expansion instead of " partial " expansion, the method just described would not be applicable. For instance, if the actual exhaust pressure were 20 lbs., the work of exhaust would be 20/17 of O–b, being represented by b–c'. This work is constant regardless of final quality, but it is the work of exhausting not only the one pound of steam but in addition whatever weight of steam may have entered upon and compressed that one pound prior to its expulsion. As there is a variable weight of steam to be exhausted, the line g–g does not truly represent its internal energy at any point above point a, and no meaning attaches to its point of intersection with w–w. The actual conditions are those of compression of vapor by vapor, accompanied and complicated further by re-vaporization. An analysis might be of interest, but it would involve tedious work and description and will not be undertaken here.

CHAPTER XVI

COMPOUND OR MULTI-STAGE EXPANSION

Multi-Stage Expansion of Air or Gas. — This process is in general the reverse of compound or multi-stage compression. However, the compression of air or gases is nearly always from temperatures near atmospheric or higher; and it is practicable to transfer part of the heat generated in compression to water circulated through the cylinder jackets, the cost of the water and its circulation being comparatively low. In expansion, on the other hand, the temperatures usually are such that they can be affected but little by normal conduction of heat to the expanding air or vapor from outside substances, such as the external atmosphere or water of ordinary temperature in a cylinder jacket; and any heat supplied by a steam jacket or similar means has a very considerable cost. Therefore, as it is not practicable to supply heat during the actual expansion and as the final temperature is likely to be near to the adiabatic, often quite low, re-heating between stages is a necessity where the initial temperature is low and the expansion ratio high or even moderate. The heat-work diagrams are simple; but, of course, there are many modifying influences to be taken into account, such as pressure drop at valves, modified exhaust pressures, volume variations, varying exhaust pressures, timing of valve operation, fall in temperature by work of exhaust displacement, etc. All of these being applications of principles and methods already described, no additional examples appear necessary.

Multi-Stage Expansion of Steam. — This involves many matters of practical design-detail pertaining to valve setting, types of valves, capacities of inter-stage passages, cylinder ratios, expansion ratios, partial expansion, over-expansion, re-heating, inter-stage condensation, bleeding, re-vaporization, etc., which may not be discussed here. It is generally difficult, sometimes

impossible, to define with absolute accuracy the governing conditions for any particular stage preparatory to construction of a heat-work diagram, and to a great extent one must rely on estimates for the several values. The conditions having been determined, the diagrams are easy to construct. We shall give one example.

Heat-Work Diagram, Triple-Expansion Engine. — Let steam be used in a triple-expansion engine, starting in the first (high-pressure) cylinder at 170 lbs. absolute pressure and 400° temperature. We shall assume that the conditions for the several expansions are as follows: final pressure in first cylinder, 50 lbs. and the quality 93.9 per cent as at the adiabatic; before entrance to second cylinder, 10 per cent of the steam (by weight) is diverted for use in heating the steam in passage from second to third cylinder, and that the quality at entrance to second cylinder is 92 per cent; final pressure in second cylinder 14.7 lbs. with quality 86.2 per cent at adiabatic; between this and the third cylinder 8 per cent of the original weight of steam is taken for warming of cylinders and passages; quality at entrance to third cylinder 95.3 per cent, there having been partial re-vaporization in passage thereto, final pressure 4 lbs. with final quality 89.7 per cent at the adiabatic.

The heat-work diagrams are (A), (B), and (C) of Figure 67, each having the essential values marked thereon and being self-explanatory. Exhaust pressure is in every case assumed to be the same as the pressure at end of expansion. The stated conditions have not been chosen to represent good practice, but because they require the three types of heat-work diagram shown and described in the preceding chapter.

Assuming that there are no losses other than the 18 per cent of steam mentioned, and reading the work for each cylinder at the adiabatic discharge quality, we have for the total net work done in the engine, in B.t.u. of heat equivalent:

$$98.0 + (83.5 \times .9 + (84.0 \times .82) = 242.0 \ B.t.u.$$

As 42.45 B.t.u. per minute is the heat equivalent of the work of one horse-power, the corresponding rate per hour is

$60 \times 42.45 = 2547$ *B.t.u.;* and hence the weight of steam required per horse-power hour by our triple-expansion engine is $2547/242 = 10.5$ *lbs.* Comparing this weight with those given
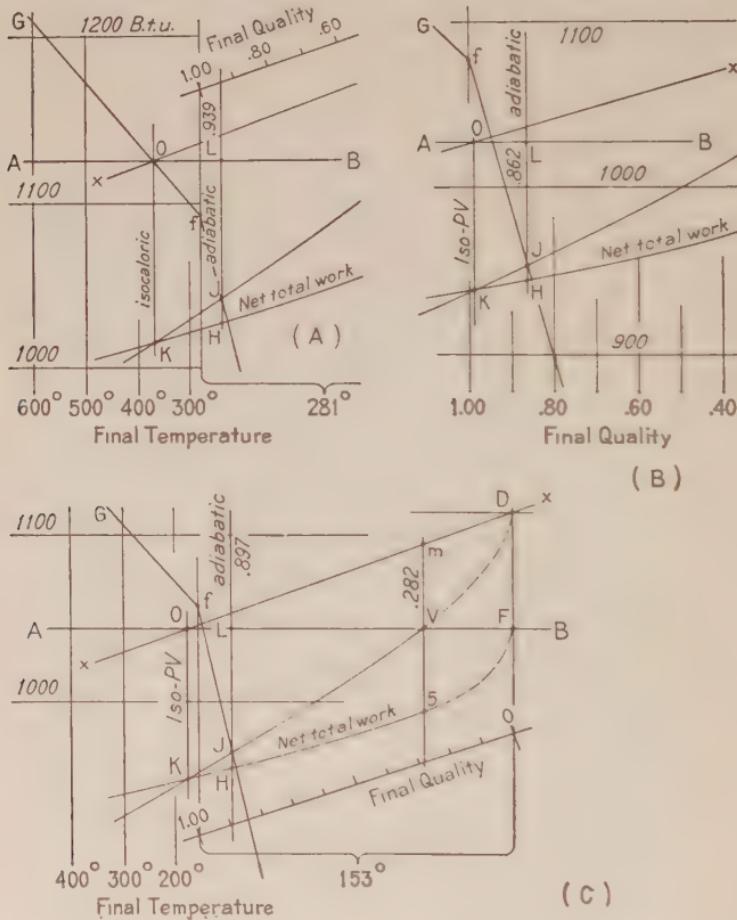
FIG. 67. HEAT-WORK DIAGRAMS. TRIPLE-EXPANSION
STEAM ENGINE

in published records of actual engine performance, we find that, as a matter of fact, triple-expansion engines require 11 to 14 lbs. of steam per indicated horse-power hour, consumption depending on the pressures used, initial temperature and the various heat losses, and details of design and operation.

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